Quantum Gravity and Scale Symmetry in Cosmology

宇宙学中的量子引力与标度对称性

C. Wetterich

C. 韦特里希

Contents

目录

C. Wetterich ( )

C. 韦特里希 ( )

Institut für Theoretische Physik, Universität Heidelberg, Heidelberg, Germany e-mail: c.wetterich@thphys.uni-heidelberg.de

德国海德堡大学理论物理研究所，电子邮箱:c.wetterich@thphys.uni-heidelberg.de

Abstract

摘要

We discuss predictions for cosmology which result from the scaling solution of functional flow equations for a quantum field theory of gravity. A scaling solution is necessary to render quantum gravity renormalizable. Our scaling solution is directly connected to the quantum effective action for the metric coupled to a scalar field. This effective action includes all effects of quantum fluctuations and is invariant under general coordinate transformations. Solving the cosmological field equations derived by variation of the quantum effective action provides for a detailed quantitative description of the evolution of the universe. The "beginning state" of the universe is found close to an ultraviolet fixed point of the flow equation. It can be described by an inflationary epoch, with approximate scale invariance of the observed primordial fluctuation spectrum explained by approximate quantum scale symmetry. Overall cosmology realizes a dynamical crossover from the ultraviolet fixed point to an infrared fixed point which is approached in the infinite future. Present cosmology is close to the infrared fixed point. It features dynamical dark energy mediated by a light scalar field. The tiny mass of this cosmon arises from its role as a pseudo-Goldstone boson of spontaneously broken quantum scale symmetry. The extremely small value of the present dark energy density in Planck units results dynamically as a consequence of the huge age of the universe. The cosmological constant problem finds a dynamical solution. We present a detailed quantitative computation of the scaling solution for the scalar effective potential and the field-dependent coefficient of the curvature scalar. This allows for further quantitative predictions.

我们讨论引力量子场论泛函流方程标度解得到的宇宙学预言。标度解是量子引力可重整化的必要条件。我们得到的标度解与耦合标量场的度规量子有效作用量直接相关。该有效作用量包含量子涨落的所有效应，且在广义坐标变换下保持不变。求解通过量子有效作用量变分得到的宇宙场方程，可以对宇宙演化给出详细定量描述。宇宙的「初态」位于流方程紫外不动点附近，可用暴胀时期描述，观测到原初涨落谱的近似标度不变性由近似量子标度对称性解释。整体来看，宇宙学实现了从紫外不动点到红外不动点的动态跨越，红外不动点将在无限远未来被趋近。当前宇宙接近红外不动点，其特征为轻标量场介导的动力学暗能量。该宇宙子的微小质量源于它作为自发破缺量子标度对称性的赝戈德斯通玻色子的性质。当前暗能量密度在普朗克单位下的极小值是宇宙年龄极大带来的动力学结果，宇宙学常数问题得到了动态解。我们给出了标量有效势和曲率标量场依赖系数标度解的详细定量计算，这可以得到更多定量预言。

Keywords

关键词

Quantum gravity · Cosmology · Asymptotic safety · Functional flow equation - Scaling solution - Crossover cosmology - Cosmological constant - Inflation - Dark energy

量子引力 · 宇宙学 · 渐近安全 · 泛函流方程 - 标度解 - 交叉宇宙学 - 宇宙学常数 - 暴胀 - 暗能量

Introduction

引言

Cosmology is a testing ground for quantum gravity. First of all, this concerns the beginning of the universe which is often associated in classical gravity to a "big bang singularity." In this very early epoch, quantum gravity effects induced by fluctuations of the metric are generally thought to be important. Very early cosmology is often described by an inflationary epoch [60,77,84,111,117] whose properties depend crucially on the shape of the potential for a scalar field. One may wonder if quantum gravity permits to compute this "inflaton potential" or, at least, some of its important qualitative properties. The shape of this potential can be tested by indirect observations of the primordial fluctuation spectrum through the observed anisotropies of the cosmic microwave background (CMB). Quantum gravity predictions for the inflaton potential can be falsified by CMB observations, constituting important tests for a given approach or model. We will see that quantum gravity may even predict the shape of the scalar potential in a field region that is relevant for present dynamical dark energy or quintessence [3, 4, 14, 24, 47, 51, 78 , 92, 102, 123, 128, 150] . In this case the detailed observations of the properties of dark energy constitute further tests of a quantum gravity model.

宇宙学是量子引力的试验场。首先，这关乎宇宙的起源，在经典引力中，宇宙起源通常与“大爆炸奇点”相关。普遍认为，在这一极早期阶段，由度规涨落诱发的量子引力效应十分重要。极早期宇宙通常可以用暴胀阶段描述 [60,77,84,111,117]，暴胀的性质关键依赖于标量场势的形状。人们不禁会问，量子引力能否计算出这个“暴胀子势”，或者至少能否计算出它的一些重要定性性质？该势的形状可以通过原初涨落谱的间接观测得到检验，观测途径是宇宙微波背景 (CMB) 的各向异性。量子引力对暴胀子势的预言可以被 CMB 观测证伪，构成了对特定方法或模型的重要检验。我们将会看到，量子引力甚至可以预言适用于当前动力学暗能量或精质的场区域中标量势的形状 [3, 4, 14, 24, 47, 51, 78 , 92, 102, 123, 128, 150] 。在这种情况下，对暗能量性质的详细观测构成了对量子引力模型的进一步检验。

In this work we focus on the formulation of quantum gravity as a quantum field theory for the metric. Further, degrees of freedom are the fields for the standard model of particle physics and beyond and a scalar singlet field that can play the role of the inflaton in early cosmology or the cosmon for late cosmology. A quantum field theory of gravity is ”renormalizable” and ”ultraviolet complete” if it admits an ultraviolet (UV) fixed point. Such a fixed point requires the existence of a ”scaling solution” for the functional flow equations describing the scale dependence of couplings or coupling functions. At a fixed point the physics becomes independent of any length scale and can therefore be extrapolated to arbitrarily short distances. If interactions are present at the UV fixed point, a theory is called ”asymptotically safe” [124]; otherwise, it is ”asymptotically free” [57,99].

本文我们聚焦于将量子引力表述为度规的量子场论。除此之外，还有粒子物理标准模型及超出标准模型部分的自由度，以及一个标量单态场——它可以在早期宇宙中充当暴胀子，或在晚期宇宙中充当宇宙子。若量子引力的量子场论存在一个紫外 (UV) 不动点，那么它就是“可重整化”且“紫外完备”的。这样的不动点要求，描述 couplings 或耦合函数标度依赖性的泛函流方程存在一个“标度解”。在不动点处，物理规律与任何长度标度无关，因此可以外推到任意短距离。若紫外不动点处存在相互作用，该理论就被称为“渐近安全”[124]；反之，它是“渐近自由”[57,99]。

The main reason for our focus is that modern functional renormalization group techniques [103, 105, 149] (see Ref. [11] for a recent review) permit a detailed quantitative computation of the flow equations for quantum gravity and, therefore, a detailed understanding of the scaling solutions. Quantum fluctuations of the metric are found to play an important role for many properties of the scalar effective potential or similar coupling functions. The effect of the metric fluctuations is described quantitatively and permits important predictions for cosmology. So far other approaches to quantum gravity do not yet yield a sufficient quantitative understanding of the effects of quantum fluctuations that would allow for a meaningful comparison with our quantitative results. Either fluctuation effects of the metric are very difficult to be incorporated, especially in the non-perturbative region relevant for fixed points. This is the case for string theories. Or it is difficult to formulate diffeomorphism invariant field equations for some type of metric field or a similar ”geometric field.” Such field equations are crucial for a quantitative description of cosmology. This present shortcoming is typically given for lattice approaches to quantum gravity. For some other approaches the issue may simply be the lack of present computational capability. Waiting for further developments of alternative approaches, we are aware that the limitations of the present work do not do justice to many interesting qualitative arguments and conjectures of these approaches for the beginning stage of the universe. We also remain strictly within the setting of a diffeomorphism invariant effective action and the field equations following from it. This restriction omits many interesting proposals for ”shortcuts” by identifying the renormalization scale k of functional flow equations with some geometrical quantity; see the review [98] and references therein.

我们聚焦于此的核心原因是，现代泛函重整化群技术 [103, 105, 149](近期综述见文献 [11]) 可以对量子引力的流方程进行详细定量计算，进而让我们详细理解标度解。我们发现，度规的量子涨落对标量有效势或类似耦合函数的诸多性质都发挥着重要作用。度规涨落的效应可以被定量描述，还能为宇宙学给出重要预言。到目前为止，其他量子引力方法仍未充分定量理解量子涨落的效应，无法和我们的定量结果进行有意义的比较。要么度规的涨落效应极难纳入，尤其是对不动点相关的非微扰区域而言，弦论就属于这种情况。要么很难对某种度规场或类似的“几何场”写出微分同胚不变的场方程，而这类场方程对宇宙学的定量描述至关重要，量子引力的格点方法通常存在这一短板。对另一些方法而言，问题可能仅仅在于目前缺乏计算能力。在等待替代方法进一步发展的同时，我们也清楚，本文的局限性未能充分体现这些方法针对宇宙起源提出的诸多有趣定性论证和猜想。我们也严格限定在微分同胚不变有效行动及其导出的场论框架内。这一限制舍去了许多有趣的“捷径”提议，这些提议将泛函流方程的重整化标度 k 等同于某个几何量，相关讨论见综述 [98] 及其中参考文献。

A central ingredient for the present work is the scaling solution for the scalar effective potential and the field- and scale-dependent ”curvature coefficient” or ”effective Planck mass” [64, 65, 91, 143, 144, 154]. Within our approximations, we find that such a scaling solution exists and permits gravity to be a renormalizable quantum field theory. We compute the scaling solutions quantitatively. The most predictive scenario is realized by ”fundamental scale invariance” [146] for which the scaling solution directly describes the quantum effective action. For a more general renormalizable quantum field theory, a small number of ”relevant parameters” describes the flow away from the UV fixed point as the renormalization scale is lowered towards the infrared. The presence of these additional free parameters reduces somewhat the predictive power for some of the quantitative results. The overall picture of cosmology remains similar, however.

本文的核心要素是标量有效势的标度解，以及依赖场和标度的“曲率系数”或“有效普朗克质量”[64, 65, 91, 143, 144, 154]。在我们的近似下，我们发现这类标度解存在，且使得引力可以成为可重整化量子场论。我们定量计算了标度解。预言性最强的场景由“基本标度不变性”[146] 实现，该情况下标度解直接描述量子有效作用量。对更一般的可重整化量子场论而言，当重整化标度向红外降低时，少量“相关参数”就可以描述流偏离紫外不动点的行为。这些额外自由参数的存在会在一定程度上降低部分定量结果的预言能力，但整体宇宙学图像仍然保持相似。

Our approach leads to several key predictions for cosmology:

我们的方法为宇宙学得出了若干关键预言:

(i) The beginning epoch of the universe can be described by inflationary cosmology.

(i) 宇宙的初始纪元可以用暴胀宇宙学描述。

(ii) The cosmological constant problem finds a dynamical solution.

(ii) 宇宙学常数问题存在一个动力学解。

(iii) The overall history of the universe is a crossover from the vicinity of a UV fixed point in the infinite past to an IR fixed point in the infinite future. The approximate quantum scale symmetry near the UV fixed point explains the approximate scale invariance of the primordial cosmic fluctuation spectrum.

(iii) 宇宙的整体演化是从无限过去紫外不动点附近向无限未来红外不动点的渡越。紫外不动点附近的近似量子标度对称性解释了原初宇宙涨落谱的近似标度不变性。

(iv) The approach to the IR fixed point realizes some form of dynamical dark energy.

(iv) 趋近红外不动点的过程实现了某种形式的动力学暗能量。

(v) The quantum scale symmetry at the IR fixed point is broken spontaneously, inducing a massless Goldstone boson. Close to the fixed point, a very small mass for the cosmon - the pseudo-Goldstone boson of spontaneously broken approximate quantum scale symmetry - is induced by explicit symmetry breaking of dilatation symmetry.

(v) 红外不动点处的量子标度对称性发生自发破缺，产生了一个无质量戈德斯通玻色子。在不动点附近，显式的标度对称性破缺给宇宙子——自发破缺近似量子标度对称性产生的赝戈德斯通玻色子——赋予了极小的质量。

(vi) The tiny ratio $U/M^4 \approx 10^{-120}$ of the present dark energy density $\sim U$ over the fourth power of the Planck mass M can find a simple explanation by the huge age of the universe in Planck units. This is similar to a similar tiny ratio for the matter energy density or radiation energy density.

(vi) 当前暗能量密度 $\sim U$ 与普朗克质量 M 四次方的微小比值 $U/M^4 \approx 10^{-120}$，可以通过宇宙在普朗克单位下的极长年龄得到简单解释，这与物质能量密度或辐射能量密度的微小比值类似。

Further, more detailed quantitative predictions for fundamental scale invariance will be developed in the main text and are summarized in the conclusions.

关于基本标度不变性更详细的定量预言将在正文中展开，并在结论中总结。

We concentrate in this work on an approximation of the exact flow equation [43, 82, 105, 121, 149] which consists in truncating the flowing effective action or effective average action for the metric and scalar field to the most general diffeomorphism invariant form containing up to two derivatives of the fields. This truncation involves three functions of the scalar field: the effective potential, the ”curvature coefficient” which multiplies the term linear in the curvature scalar R , and the ”kinetial” or ”wave function renormalization” which multiplies the kinetic term of the scalar field. Within this truncation we present explicit computations for the effective potential and the curvature coefficient, while similarly robust results for the kinetial are not yet available. The truncation to two derivatives may be expected to be valid if typical momenta are sufficiently small as compared to the effective Planck mass. This seems to be realized for the late stages of inflation relevant for the observable primordial fluctuation spectrum and for all later epochs of the universe. Towards the beginning in the early stages of inflation, terms with more than two derivatives may become more important. This holds, in particular, if quantum gravity is asymptotically free [6, 50, 110, 119] with dominant terms quadratic in the curvature tensor involving four derivatives. It is also possible that the beginning stage is better described in terms of other degrees of freedom, for example, by gauge fields and a vierbein in ”pregeometry” [145].

本文我们聚焦于精确流方程 [43, 82, 105, 121, 149] 的一种近似: 将度规和标量场的流有效作用量 (或有效平均作用量) 截断为最一般的微分同胚不变形式，仅包含不超过二阶的场导数。该截断包含标量场的三个函数: 有效势、乘以曲率标量 R 线性项的“曲率系数”，以及乘以标量场动能项的“动力学系数”(或称波函数重整化因子)。在该截断内我们给出了有效势和曲率系数的显式计算，而动力学系数尚未得到同样可靠的结果。当典型动量远小于有效普朗克质量时，二阶导数截断应当成立。对于可观测原初涨落谱相关的暴胀晚期阶段，以及宇宙所有更晚的纪元，这一条件似乎都满足。朝向暴胀初期的开端，高于二阶导数的项会变得更重要，尤其是当量子引力渐近自由 [6, 50, 110, 119] ，曲率张量二阶项 (含四阶导数) 占主导时。此外，初始阶段用其他自由度描述可能更合适，例如“前几何”[145] 中的规范场和 vierbein。

In quantum field theory the observables do not depend on the particular choice of fields used to describe them. In particular, the choice of the metric is not unique. It may be changed by multiplication with a function of the scalar field, the so-called Weyl scaling [31,155]. Different choices of the metric correspond to different ”metric frames.” The physics expressed in terms of observables is independent of the choice of the metric frame. We present many results directly in terms of frame-invariant equations [70, 139]. Nevertheless, in making contact with intuition and facilitating comparison with the existing literature, it is useful to discuss a Weyl transformation to the ”Einstein frame” for which the Planck mass takes a fixed value. This fixed Planck mass is not an intrinsic scale of the quantum gravity model, being introduced only by a field transformation. One should not be surprised that many simple findings associated to fixed points and quantum scale symmetry get obscured if an ”artificial mass scale” is introduced. This explains why the naturalness of some of our results is not easily seen in the Einstein frame, for which too simple estimates would judge them as unnatural. This concerns, in particular, the properties of the IR fixed point which lead to an effective potential that vanishes naturally for large field values and the associated very small mass of the scalar field responsible for dynamical dark energy.

量子场论中，可观测量不依赖于描述它们所用的特定场选择，尤其度规的选择不是唯一的。度规可以通过乘以标量场的函数改变，这就是外尔标度 [31,155]。不同的度规选择对应不同的“度规规范”。用可观测量表述的物理与度规规范的选择无关。我们许多结果直接以规范不变方程的形式给出 [70, 139]。尽管如此，为契合直觉、方便与现有文献比较，讨论到“爱因斯坦规范”的外尔变换是有用的，在爱因斯坦规范中普朗克质量取固定值。这个固定普朗克质量不是量子引力模型的内禀标度，只是通过场变换引入的。如果引入一个“人工质量标度”，不动点和量子标度对称性相关的诸多简单结论都会变得模糊，这并不奇怪。这也解释了为何我们部分结果的自然性在爱因斯坦规范中不易显现——在爱因斯坦规范中，过于简单的估计会判定它们不自然。这一点尤其适用于红外不动点的性质: 红外不动点使得大场取值下有效势自然趋于零，且负责动力学暗能量的标量场质量天然极小。

This work is organized as follows: In section ”Cosmology Beyond Einstein Gravity” we briefly discuss the need for cosmology beyond Einstein gravity which motivates to consider a scalar field along with the metric. Section ”Quantum Gravity” turns to the formulation of quantum gravity as a quantum field theory for the metric and a scalar field. It explains basic notions as the flow equations and the scaling solution. It also presents first qualitative results on a simple level. In section ”Quantum Scale Symmetry” we emphasize the crucial role of quantum scale symmetry for the flow close to the UV and IR fixed points. This provides already for an overall picture of cosmology as a crossover from the UV to the IR fixed point, connecting an early inflationary epoch to late cosmology with dynamical dark energy. Section ”Flow Equations for Quantum Gravity” is devoted to quantitative results for the flow equations and we discuss the corresponding scaling solution in section ”Scaling Solution.” On this basis we discuss the crossover cosmology associated to the scaling solution in more quantitative detail in section ”Crossover Cosmology.” We describe how the sequence of different epochs in the evolution of the universe emerges naturally for our setting of quantum gravity. In section ”Fundamental Scale Invariance” we focus on fundamental scale invariance, shedding more light on which predictions arise only for this particular setting while more freedom is left for a general renormalizable quantum field theory of gravity. In section ”Discussion” we summarize our results.

本文结构安排如下: 在“超越爱因斯坦引力的宇宙学”一节中，我们简要讨论了超越爱因斯坦引力研究宇宙学的必要性，这也是我们在度规之外引入标量场的动机。“量子引力”一节转而将量子引力表述为度规和标量场的量子场论，阐述了流方程、标度解等基本概念，并在简单层面给出了初步定性结果。在“量子标度对称性”一节中，我们强调了量子标度对称性对于紫外和红外不动点附近流的关键作用，由此已经可以得到宇宙学从紫外不动点到红外不动点穿越的整体图像，该图像将早期暴胀时期与含动力学暗能量的晚期宇宙学联系了起来。“量子引力的流方程”一节专门介绍流方程的定量结果，我们在“标度解”一节讨论了对应的标度解。在此基础上，我们在“穿越宇宙学”一节更定量地详细讨论了与标度解相关的穿越宇宙学，阐述了在我们的量子引力框架下，宇宙演化中不同时期的序列如何自然涌现。在“基本标度不变性”一节中，我们聚焦于基本标度不变性，进一步阐明哪些预测仅适用于该特定框架，而广义可重整化引力量子场论则存在更多自由度。我们在“讨论”一节总结了本文的研究结果。

Cosmology Beyond Einstein Gravity

超越爱因斯坦引力的宇宙学

General relativity or Einstein gravity is a classical field theory, based on the Einstein-Hilbert action

广义相对论 (即爱因斯坦引力) 是一种经典场论，其基础是爱因斯坦-希尔伯特作用量

$$S = \int_x \sqrt{g}\left\{-\frac{M^2}{2}R + V\right\}. \tag{1}$$

Here R is the curvature scalar formed from the metric $g_{\mu\nu}(x)$ and $g = \det\left(g_{\mu\nu}\right)$. (The factor $\pm i$ for $g < 0$ plays no role for the field equations.) The integral $\int_x$ is an integral over four-dimensional space. This theory involves two parameters, the (reduced) Planck mass $M = 2.436 \cdot 10^{18}$GeV and the cosmological constant $V = \left(2 \cdot 10^{-3}\mathrm{eV}\right)^4$. Gravity is characterized by a fundamental symmetry, namely, diffeomorphism symmetry, which is equivalent to invariance under general coordinate transformations. Infinitesimal diffeomorphism transformations can be formulated as variations of the metric at fixed coordinates, with infinitesimal parameters $\xi^\mu(x)$,

其中 R 是由度规 $g_{\mu\nu}(x)$ 和 $g = \det(g_{\mu\nu})$ 构造的曲率标量。(因子 $\pm i$ 对 $g < 0$ 而言在场方程中不起作用。) 积分 $\int_x$ 是对四维空间的积分。该理论包含两个参数: 约化普朗克质量 $M = 2.436 \cdot 10^{18}\mathrm{GeV}$ 和宇宙学常数 $V = \left(2 \cdot 10^{-3}\mathrm{eV}\right)^4$ 。引力具有一个基本对称性，即微分同胚对称性，它等价于广义坐标变换下的不变性。无穷小微分同胚变换可以表述为固定坐标下度规的变分，其无穷小参数为 $\xi^\mu(x)$,

$$\delta g_{\mu\nu} = -\partial_\mu \xi^\rho g_{\rho\nu} - \partial_\nu \xi^\rho g_{\mu\rho} - \xi^\rho \partial_\rho g_{\mu\nu}. \tag{2}$$

The gravitational field equations are obtained by variation of S with respect to $g_{\mu\nu}(x)$.

引力场方程可通过对 $g_{\mu\nu}(x)$ 变分 S 得到。

Together with a minimal coupling of the metric to matter fields, both through covariant derivatives and the overall factor $\sqrt{g}$, Einstein gravity is an extremely successful model for most observations in gravity and cosmology. It describes the gravity of stars and black holes, as well as galaxies or larger structures once some form of dark matter is included. It is tested with high precision by experiments from the submillimeter scale to the size of our solar system. For a suitable matter content of the universe, Einstein gravity successfully describes the hot radiation-dominated universe and the subsequent matter-dominated universe, as tested by nucleosynthesis or the cosmic microwave background radiation (CMB). In the presence of the cosmological constant V , it can account for dark energy and the associated accelerated expansion in the recent cosmological epoch.

通过将度规以最小耦合方式与物质场结合——同时通过协变导数和整体因子 $\sqrt{g}$ ——爱因斯坦引力是一个极为成功的模型，能够解释绝大多数引力与宇宙学观测结果。它描述了恒星和黑洞的引力，在引入某种暗物质后也能解释星系及更大尺度结构的引力。它已经从亚毫米尺度到太阳系尺度得到了高精度的实验检验。对于宇宙合适的物质组分，爱因斯坦引力可以成功描述热辐射主导的宇宙以及之后的物质主导宇宙，这已经被核合成和宇宙微波背景辐射 (CMB) 观测所证实。在存在宇宙学常数 V 的情况下，它可以解释暗能量以及近代宇宙学时期对应的加速膨胀。

Nevertheless, Einstein gravity has also a few important shortcomings.

尽管如此，爱因斯坦引力仍存在一些重要缺陷。

(i) The classical theory has to be extended to quantum gravity in order to permit a consistent coupling to quantum matter.

(i) 经典理论必须扩展为量子引力，才能实现与量子物质的自洽耦合。

(ii) The tiny dimensionless ratio $V/M^4 \approx 10^{-120}$ remains unexplained.

(ii) 微小的无量纲比 $V/M^4 \approx 10^{-120}$ 仍未得到解释。

(iii) The initial value for the energy density must be extremely close to the critical density, which needs tremendous fine-tuning.

(iii) 能量密度的初始值必须极度接近临界密度，这需要极大的精细调谐。

(iv) The high degree of isotropy of the CMB remains a mystery, since within Einstein gravity the radiation from different directions (angles larger than $\sim 1^\circ$) is emitted from regions that never had causal contact.

(iv) CMB 的高度各向同性仍然是一个谜题，因为在爱因斯坦引力框架下，不同方向 (角度大于 $\sim 1^\circ$) 的辐射来自从未有过因果接触的区域。

(v) The singularities in the center of the black hole or at the beginning of the universe may point to an incompleteness of this theory.

(v) 黑洞中心或宇宙开端的奇点可能暗示了该理论的不完备性。

(vi) A two-parameter model for dark energy is very predictive and may be falsified if the present observational tensions (Hubble tension, etc.) evolve to hard contradictions.

(vi) 暗能量的两参数模型预言性很强，如果当前的观测张力 (哈勃张力等) 演变为确实的矛盾，该模型就可能被证伪。

Many (if not all) of the shortcomings may be cured by a quantum field theory for the metric coupled to a scalar field χ . This scalar field is a singlet with respect to the gauge group of the standard model. It can play the role of the inflaton for an early inflationary epoch of the universe [60, 77, 84, 111, 117] . Inflation can explain the closeness to the critical energy density and many properties of the primordial fluctuations which are observed through the anisotropies of the CMB. For the present cosmology, the scalar field can be associated to the cosmon, a very light scalar field responsible for dynamical dark energy [3, 4, 14, 24, 47, 51, 78, 92, 102, 123, 128, 150] and associated to a dynamical explanation for the tiny ratio between the dark energy density and M^4 around 10^{-120} . It is possible that the same scalar field accounts for inflation and dynamical dark energy [9, 13, 28, 32, 56, 59, 66 − 68, 93, . A quantum field theory includes quantum fluctuations of both the scalar and the metric field. Generalizations may replace the metric by the vierbein or introduce other "pregeometric" fields [145]. We remain for this note with the metric. We also do not discuss the interesting possibility of asymptotically free quantum gravity which involves higher-order curvature invariants [6, 50, 110, 119] .

诸多 (即便不是全部) 缺陷都可以通过度量耦合标量场 χ 的量子场论解决。该标量场是标准模型规范群的单态，可充当宇宙早期暴涨阶段的暴涨子 [60, 77, 84, 111, 117] 。暴涨能够解释宇宙为何接近临界能量密度，以及通过宇宙微波背景各向异性观测到的原初涨落的诸多性质。对于现代宇宙学，该标量场可对应宇宙子——这是一种极轻的标量场，是动力学暗能量 [3, 4, 14, 24, 47, 51, 78, 92, 102, 123, 128, 150] 的来源，也为暗能量密度与 M^4 之间约为 10^{-120} 的微小比值提供了动力学解释。同一个标量场有可能同时解释暴涨与动力学暗能量 [9, 13, 28, 32, 56, 59, 66 − 68, 93, 94, 108, 116, 133] 。量子场论包含标量场与度量场的量子涨落。推广后的理论可以用双四元矢代替度量，或引入其他“前几何”场 [145]。本文我们仍使用度量，也不讨论渐近自由量子引力这一涉及高阶曲率不变量的有趣可能性 [6, 50, 110, 119] 。

The central quantity for discussing cosmology in a quantum field theory is the quantum effective action $\Gamma\left[g_{\mu\nu}, \chi\right]$. Here $g_{\mu\nu}(x)$ corresponds to the expectation value of the fluctuating metric field, and χ is the expectation value of a fluctuating scalar field. Formally, the functional $\Gamma\left[g_{\mu\nu}, \chi\right]$ generates the one-particle irreducible Green’s functions. The first variation of Γ with respect to $g_{\mu\nu}$ or χ yields the exact quantum field equations, possibly with a source term arising from the matter part that we do not discuss here explicitly. These field equations constitute the relevant evolution equations for cosmology. The second functional variation yields the inverse correlation function. The correlation functions for the scalar and the metric field encode the primordial scalar and tensor fluctuations [139, 141]. In order to establish the direct connection between Γ and the observable quantities, a diffeomorphism invariant form of the quantum effective action is needed [142]. The aim of quantum gravity is the computation of Γ by including all effects of fluctuations of the metric and other fields.

> 在量子场论中讨论宇宙学的核心量是量子有效作用量 $\Gamma\left[g_{\mu\nu},\chi\right]$ 。此处 $g_{\mu\nu}(x)$ 对应涨落度量场的期望值，χ 是涨落标量场的期望值。形式上，泛函 $\Gamma\left[g_{\mu\nu},\chi\right]$ 生成单粒子不可约格林函数。Γ 对 $g_{\mu\nu}$ 或 χ 的一阶变分给出精确量子场方程，方程可能包含物质部分带来的源项，本文不对此展开讨论。这些场方程构成了宇宙学对应的演化方程。二阶泛变分给出逆关联函数。标量场与度量场的关联函数包含了原初标量涨落与张量涨落的信息 [139, 141]。为了建立 Γ 与可观测量之间的直接联系，需要微分同胚不变形式的量子有效作用量 [142]。量子引力的目标就是纳入度量与其他所有场涨落的效应，计算得到 Γ 。

Before reporting on progress for the computation of Γ , it is useful to discuss a few general properties. Diffeomorphism symmetry requires the invariance of Γ with respect to the transformation (2), combined with $\delta\chi = -\xi^{\rho}\partial_{\rho}\chi$. We also assume a discrete symmetry $\chi \to -\chi$. For low enough (covariant) momenta and small enough curvature invariants, one expects the validity of a derivative expansion. Up to second order in the derivatives, the effective action in the gravity-scalar sector takes the form

> 在介绍 Γ 计算的研究进展之前，先讨论几个一般性质十分必要。微分同胚对称性要求 Γ 在变换 (2) 下不变，且结合满足 $\delta\chi = -\xi^{\rho}\partial_{\rho}\chi$ 。我们还假设存在离散对称性 $\chi \to -\chi$ 。对于足够低的协变动量与足够小的曲率不变量，一般认为导数展开成立。到导数二阶项为止，引力-标量 sector 的有效作用量形式为

$$\Gamma = \int_{x}\sqrt{g}\left\{-\frac{1}{2}F(\chi)R + \frac{1}{2}K(\chi)\partial^{\mu}\chi\partial_{\mu}\chi + U(\chi)\right\}. \tag{3}$$

This has to be supplemented by the effective action for the fields of the standard model of particle physics. The effective scalar potential $U(\chi)$ generalizes the cosmological constant V . In the presence of other scalar fields as the Higgs scalar, this potential is supposed to describe the relative minimum with respect to the other fields. The ”kinetial” $K(\chi)$ multiplies the scalar kinetic term, with $\partial^{\mu}\chi = g^{\mu\upsilon}\partial_{\upsilon}\chi$ and $g^{\mu\upsilon}g_{\upsilon\rho} = \delta^{\mu}_{\rho}$. Finally, the curvature coefficient $F(\chi)$ generalizes M^2 . Its χ -dependence yields a modification of general relativity which may be named ”variable gravity” [136].

> 它必须辅以粒子物理标准模型中各场的有效作用量。有效标量势 $U(\chi)$ 是对宇宙学常数 V 的推广。当存在希格斯标量场这类其他标量场时，该势可描述该势相对于其他场的相对极小值。“动能项” $K(\chi)$ 乘以标量动力学项，其中包含 $\partial^{\mu}\chi = g^{\mu\upsilon}\partial_{\upsilon}\chi$ 和 $g^{\mu\upsilon}g_{\upsilon\rho} = \delta^{\mu}_{\rho}$ 。最后，曲率系数 $F(\chi)$ 是对 M^2 的推广。它对 χ 的依赖给出了广义相对论的一种修正形式，可称之为“可变引力”[136]。

Unless dimensionless coefficients of higher-derivative invariants as $R^2, R^{\mu\upsilon}R_{\mu\upsilon}$, $\left(\partial^{\mu}\chi\partial_{\mu}\chi\right)^2/F^4$, etc. are much larger than one, the effective action (3) provides for a good approximation to the epoch of inflation relevant for the observed fluctuations, as well as for all later cosmology. Furthermore, a rather large class of modified gravity theories with higher derivatives can be brought to the form (3) by using appropriate fields and variable transformations [153]. This includes Starobinsky inflation [117] which involves a very large coefficient of the higher-derivative invariant R^2 .

除非 $R^2, R^{\mu\upsilon}R_{\mu\upsilon}$、$\left(\partial^\mu\chi\partial_\mu\chi\right)^2/F^4$ 等高阶导数不变量的无量纲系数远大于 1，否则有效作用量 (3) 对与观测涨落相关的暴胀时期以及所有后续宇宙学演化都能给出良好近似。此外，相当一大类含高阶导数的修正引力理论都可以通过合适的场选择与变量变换化为形式 (3)[153]，其中就包括 Starobinsky 暴胀 [117]，该模型中高阶导数不变量 R^2 的系数非常大。

The field equations obtained by variation of the effective action for variable gravity (3) can be found in Ref. [136]. We are mainly interested here in homogeneous and isotropic solutions which correspond to a scalar field $\chi(\eta)$ only depending on conformal time η and a metric $g_{\mu\nu} = a^2(\eta)\eta_{\mu\nu}$, with $a(\eta)$ the cosmic scale factor and $\eta_{\mu\nu} = \mathrm{diag}(-1,1,1,1)$. It is convenient to introduce

可变引力有效作用量 (3) 经变分得到的场方程可见文献 [136]。本文我们主要关注均匀各向同性解，对应标量场 $\chi(\eta)$ 仅依赖共形时间 η，度规取 $g_{\mu\nu} = a^2(\eta)\eta_{\mu\nu}$ 形式，其中 $a(\eta)$ 是宇宙标度因子，且满足 $\eta_{\mu\nu} = \mathrm{diag}(-1,1,1,1)$。引入以下量会更为方便:

$$A = \sqrt{F}a,\ \hat{\mathcal{H}} = \partial_\eta \ln A = \mathcal{H} + \frac{1}{2}\partial_\eta \ln F, \tag{4}$$

with the conformal Hubble parameter $\mathcal{H} = \partial_\eta \ln a = Ha$ related to the Hubble parameter $H = \partial_t \ln a$ and cosmic time t related to η by $\mathrm{d}t = a\ \mathrm{d}\eta$. We can write [139] the homogeneous gravitational field equations for spatially flat geometries in the simple form

其中共形哈勃参数 $\mathcal{H} = \partial_\eta \ln a = Ha$ 与哈勃参数 $H = \partial_t \ln a$ 相关，宇宙时间 t 与 η 满足关系 $\mathrm{d}t = a\ \mathrm{d}\eta$。我们可以 [139] 将空间平坦几何下的均匀引力场方程写为如下简单形式:

$$2\hat{\mathcal{H}}^2 + \partial_\eta\hat{\mathcal{H}} = A^2\hat{V},\ \hat{\mathcal{H}}^2 - \partial_\eta\hat{\mathcal{H}} = \frac{\hat{K}}{2}\left(\partial_\eta\chi\right)^2, \tag{5}$$

while the scalar field equation reads

而标量场方程为:

$$\hat{K}\left(\partial_\eta^2 + 2\hat{\mathcal{H}}\partial_\eta\right)\chi + \frac{1}{2}\frac{\partial\hat{K}}{\partial\chi}\left(\partial_\eta\chi\right)^2 + A^2\frac{\partial\hat{V}}{\partial\chi} = 0. \tag{6}$$

As usual, only two of the three equations (5), (6) are independent.

和通常情况一样，(5)、(6) 这三个方程中只有两个是独立的。

These equations employ the combinations

这些方程用到了以下组合:

$$\hat{V} = \frac{U}{F^2},\ \hat{K} = \frac{K}{F} + \frac{3}{2F^2}\left(\frac{\partial F}{\partial\chi}\right)^2. \tag{7}$$

Out of the three functions F, K , and U , only the two particular combinations $\hat{V}$ and $\hat{K}$ matter. We will later see that Eqs. (5)-(7) are valid in all metric frames related by conformal or Weyl scalings, with frame-invariant combinations $\widehat{V}$ and $\widehat{K}$. For the special case $F = M^2, K = 1$, they reduce to the well-known cosmological equations

在三个函数 F, K 和 U 中，只有 $\widehat{V}$ 和 $\widehat{K}$ 这两个特定组合起作用。我们在后文会看到，式 (5)-(7) 在所有经共形标度或外尔标度联系的度规框架中都成立，其中 $\widehat{V}$ 和 $\widehat{K}$ 就是框架不变量组合。对于特殊情况 $F = M^2, K = 1$ ，它们会退化为人们熟知的宇宙学方程:

$$H^2 = \frac{1}{3M^2}\left[U + \frac{1}{2}(\partial_t \chi)^2\right]$$

$$\partial_t^2 \chi + 3H\partial_t \chi = -\frac{\partial U}{\partial \chi} \tag{8}$$

Quantum Gravity

量子引力

The usual approach to cosmology beyond Einstein gravity assumes a particular form of the functions $U(\chi), F(\chi)$, and $K(\chi)$, solves the field equations, and discusses consequences for observations. It should be the aim of a theory of quantum gravity to compute these functions, or at least to restrict their qualitative behavior. The transition from assumption to computation can be a major step in our understanding of cosmology. Using functional flow equations [12, 103, 105, 149], we will find that the fluctuations of the metric indeed imply important restrictions and new features for the shape of the functions U, F, K . These effects are typically non-perturbative. It would be very important to compare these qualitative features with other approaches to quantum gravity. This would require other methods that can cope with the effects of metric fluctuations, including the non-perturbative domain. Unfortunately, such alternative methods seem not yet to be available.

超越爱因斯坦引力的常规宇宙学方法会假设函数 $U(\chi), F(\chi)$ 和 $K(\chi)$ 具有特定形式，求解场方程，并讨论观测结果。量子引力理论的目标应当是计算这些函数，或至少限制它们的定性行为。从假设到计算的转变，是我们理解宇宙学过程中的重要一步。利用泛函流方程 [12, 103, 105, 149] 我们将发现，度规涨落确实对函数 U, F, K 的形式给出了重要限制，也带来了新特性。这些效应通常是非微扰的。将这些定性特征与其他量子引力方法进行比较非常重要，这需要其他能够处理度规涨落效应 (包括非微扰区域效应) 的方法。遗憾的是，这类替代方法目前似乎尚不存在。

Flow Equation

流方程

A complete model of quantum gravity should be valid for all distance or momentum scales. If it does not involve a fundamental smallest length, the model should describe what happens as length scales approach zero or momenta reach infinity. The physics at different momentum scales typically changes due to the effects of quantum fluctuations. Effective laws at a given momentum-type scale k may be encoded in a scale-dependent effective action Γ_k . More precisely, we define Γ_k by only including the quantum fluctuations with squared (covariant) momenta $|q^2| \gtrsim k^2$. In the limit $k \to 0$, all fluctuations are included, such that for $k \to 0$ the scale-dependent effective action Γ_k equals the quantum effective action. In the opposite limit $k \to \infty$, no fluctuations are included, and Γ_k approaches the microscopic (”classical”) action that is used to define the functional integral for a quantum field theory. If it is possible to find a valid form of the functional Γ_k for the whole range of k from infinity to zero, a quantum field theory can be considered to be complete. In this case the model defined at an arbitrary small microscopic length scale $k^{-1} \to 0$ can be related to the observable ”macrophysics” for $k \to 0$.

完整的量子引力模型应对所有距离尺度或动量尺度都成立。若模型不涉及基本最小长度，就应当描述长度尺度趋近零、动量趋近无穷时的物理行为。由于量子涨落效应，不同动量尺度的物理通常会发生变化。给定动量类尺度 k 下的有效规律可以编码在依赖尺度的有效作用量 Γ_k 中。更准确地说，我们定义 Γ_k 时仅包含协变动量平方为 $|q^2| \gtrsim k^2$ 的量子涨落。在极限 $k \to 0$ 下，所有涨落都被包含进来，因此对于 $k \to 0$ ，依赖尺度的有效作用量 Γ_k 等于量子有效作用量。在相反的极限 $k \to \infty$ 下，不包含任何涨落，Γ_k 趋近于用来定义量子场论泛函积分的微观（“经典”）作用量。若能在 k 从无穷大到零的整个范围找到泛函 Γ_k 的有效形式，就可以认为该量子场论是完备的。这种情况下，在任意小的微观长度尺度 $k^{-1} \to 0$ 定义的模型可以和 $k \to 0$ 下可观测的“宏观物理”联系起来。

The k -dependence of Γ_k is described by a functional flow equation

Γ_k 对 k 的依赖关系由泛函流方程描述

$$k\partial_k\Gamma_k = \zeta_k \tag{9}$$

where the flow generator ζ_k is a functional of $g_{\mu\nu}$ and χ . It typically involves an integral over the momenta of the fluctuations. We will work here with Euclidean momenta $q^2 \geq 0$, with analytic continuation to Minkowski signature done at the end. Since in the step from $k + \mathrm{d}k$ to k only a small range of additional fluctuations is included, this momentum integral is finite, centered around $q^2 \approx k^2$. We will employ a particular form of the flow equation based on a gauge-invariant formulation of the effective average action [142, 146]. The contribution of low-momentum fluctuations is removed by a smooth infrared cutoff function $R_k\left(q^2\right)$ which vanishes rapidly for $q^2 \gg k^2$. Besides its dependence on the infrared cutoff, the flow generator ζ_k only involves $\Gamma_k^{(2)}\left[g_{\mu\upsilon}, \chi\right]$, the second functional derivative of Γ_k evaluated for arbitrary fields $g_{\mu\upsilon}$ and χ . Thus, both sides of Eq. (9) are functionals of these fields. The functional flow equation for the effective average action is an exact identity [149]. For practical purposes it has to be approximated by truncating the most general form of Γ_k , for example, to the form (3). We will present more details in section ”Flow Equations for Quantum Gravity.” Here we first address the most important features and results.

其中流生成元 ζ_k 是 $g_{\mu\nu}$ 和 χ 的泛函。它通常包含对涨落动量的积分。我们在此处讨论欧几里得动量 $q^2 \geq 0$ ，最后再做解析延拓得到闵氏号差。由于从 $k + \mathrm{d}k$ 到 k 的步骤中仅额外包含了一小范围的涨落，该动量积分是有限的，中心位于 $q^2 \approx k^2$ 。我们将采用流方程的一种特殊形式，它基于有效平均作用量的规范不变表述 [142, 146]。低动量涨落的贡献由光滑红外截断函数 $R_k\left(q^2\right)$ 移除，该函数在 $q^2 \gg k^2$ 处快速趋于零。除了对红外截断的依赖，流生成元 ζ_k 仅涉及 $\Gamma_k^{(2)}\left[g_{\mu\upsilon}, \chi\right]$ ，也就是 Γ_k 对任意场 $g_{\mu\upsilon}$ 和 χ 的二阶泛函导数。因此，式 (9) 的两边都是这些场的泛函。有效平均作用量的泛函流方程是一个精确恒等式 [149]。出于实用考虑，需要通过截断 Γ_k 的最一般形式 (例如截断为形式 (3)) 来对它做近似。我们会在“量子引力的流方程”一节给出更多细节，在这里我们首先介绍最重要的特征和结论。

Scaling Solution

标度解

For a scaling solution Γ_k becomes independent of k once it is expressed in terms of suitable dimensionless renormalized fields and coupling functions. In our approximation this means that the dimensionless functions

对标度解而言，一旦 Γ_k 用合适的无量纲重整化场和耦合函数表示，就会变得与 k 无关。在我们的近似下，这意味着无量纲函数

$$u(\tilde{\rho}) = \frac{U}{k^4},\ f(\tilde{\rho}) = \frac{F}{k^2},\ K(\tilde{\rho}), \tag{10}$$

depend only on the dimensionless combination

仅依赖于无量纲组合

$$\tilde{\rho} = \frac{\chi^2}{2k^2} \tag{11}$$

For a scaling solution the effective average action expressed in terms of these functions solves the flow equation for the whole range of $\tilde{\rho}$ from zero to infinity. In other words, the (truncated) flow equation (9) should admit a solution for which the only k -dependence arises implicitly through the expressions (10),(11), without any additional explicit k -dependence.

对标度解来说，用这些函数表示的有效平均作用量在 $\tilde{\rho}$ 从零到无穷的整个范围内都满足流方程。换言之，(截断后的) 流方程 (9) 应当存在这样一个解: 该解仅通过式 (10)、(11) 隐式包含对 k 的依赖，不存在额外的显式 k 依赖。

The existence of a scaling solution for Γ_k implies that quantum gravity can be formulated as a complete quantum field theory. Indeed, for any finite nonzero χ we can extrapolate Γ_k arbitrarily far to the ultraviolet, $k \to \infty$, by taking the limit $\tilde{\rho} \to 0$. The infrared limit $k \to 0$ corresponds to $\tilde{\rho} \to \infty$.

Γ_k 存在标度解意味着量子引力可以构造为一个完备量子场论。实际上，对任意非零有限的 χ ，我们都可以通过取极限 $\tilde{\rho} \to 0$ 将 Γ_k 任意外推到紫外区 $k \to \infty$ ，红外极限 $k \to 0$ 对应 $\tilde{\rho} \to \infty$ 。

If $u(\tilde{\rho})$ is analytic at $\tilde{\rho} = 0$, the scaling solution corresponds to fixed points for infinitely many dimensionless renormalized couplings. We may define those couplings by a Taylor expansion of $u(\tilde{\rho})$ at $\tilde{\rho} = 0$,

若 $u(\tilde{\rho})$ 在 $\tilde{\rho} = 0$ 处解析，则标度解对应无穷多个无量纲重整化耦合的不动点。我们可以通过对 $u(\tilde{\rho})$ 在 $\tilde{\rho} = 0$ 处做泰勒展开来定义这些耦合，

$$u(\tilde{\rho}) = u_0 + \tilde{m}_0^2\tilde{\rho} + \frac{1}{2}\lambda_0\tilde{\rho}^2 + \frac{1}{6}\tilde{\gamma}_0\tilde{\rho}^3 + \dots,$$

$$U(\chi) = u_0 k^4 + \frac{\widetilde{m}_0}{2} k^2 \chi^2 + \frac{\lambda_0}{8} \chi^4 + \frac{\tilde{\gamma}_0}{48k^2} \chi^6 + \ldots \tag{12}$$

For a scaling solution the flow of $u_0, \widetilde{m}_0^2, \lambda_0, \tilde{\gamma}_0, \ldots$ becomes independent of k . These considerations extend to other functions that characterize the scaling solution for the functional Γ_k . In simple words, nothing changes anymore if the ultraviolet limit $k \to \infty$ is formulated in terms of renormalized dimensionless couplings. The theory is then ultraviolet complete. The existence of an ultraviolet fixed point at nonzero couplings is the basic idea of asymptotic safety for quantum gravity [38, 75, 103, 104, 115, 124].

对标度解而言，$u_0, \widetilde{m}_0^2, \lambda_0, \tilde{\gamma}_0, \ldots$ 的流与 k 无关。这些讨论可以推广到表征泛函 Γ_k 标度解的其他函数。简单来说，若用重整化无量纲耦合表述紫外极限 $k \to \infty$ ，就不会再有任何变化，此时理论是紫外完备的。非零耦合处存在紫外不动点是量子引力渐近安全的核心思想 [38, 75, 103, 104, 115, 124]。

We emphasize that the existence of a scaling solution does not require that the functions $u(\tilde{\rho}), f(\tilde{\rho}), K(\tilde{\rho})$ remain all finite for $\tilde{\rho} \to 0$. We have formulated the flow equation for one given field basis $\left(g_{\mu\nu}, \chi\right)$. It is possible that in this field basis some functions, say $K(\tilde{\rho})$, diverges for $\tilde{\rho} \to 0$. By nonlinear field transformations one may find a different choice of fields ("different metric frame") for which the dimensionless functions remain finite for $k \to \infty$. For the existence of a scaling solution, it is sufficient that one choice of fields exists for which Γ_k is well defined for all field values and shows no explicit k -dependence.

我们需要强调，标度解的存在并不要求函数 $u(\tilde{\rho}), f(\tilde{\rho}), K(\tilde{\rho})$ 在 $\tilde{\rho} \to 0$ 处全部保持有限。我们是针对一个给定场基 $\left(g_{\mu\nu}, \chi\right)$ 建立的流方程。在该场基中，可能存在某些函数 (例如 $K(\tilde{\rho})$) 在 $\tilde{\rho} \to 0$ 处发散。通过非线性场变换，我们可以找到另一组场的选择 (“不同度量框架”)，使得无量纲函数在 $k \to \infty$ 处仍然保持有限。对标度解的存在而言，只要存在一组场选择，使得 Γ_k 对所有场值都定义良好，且不存在显式 k 依赖就足够了。

Scaling Potential and Curvature Coefficient

标度势与曲率系数

As a central result [41, 64, 65, 97, 144] , the scaling solution for the dimensionless effective potential approaches finite constants both for $\tilde{\rho} \to 0$ and $\tilde{\rho} \to \infty$ (for details, see section "Flow Equations for Quantum Gravity"):

作为核心结果 [41, 64, 65, 97, 144] ，无量纲有效势的标度解在 $\tilde{\rho} \to 0$ 和 $\tilde{\rho} \to \infty$ 两种情况下都趋近于有限常数 (详见“量子引力流方程”一节):

$$u(\tilde{\rho} \to 0) = u_0,\ u(\tilde{\rho} \to \infty) = u_\infty. \tag{13}$$

For a given setting of flow equations and a given choice of the infrared regulator R_k , the constants u_0 and u_∞ only depend on the numbers of effectively massless particles (scalars, fermions, gauge bosons, graviton). If for $\tilde{\rho} \to \infty$ only the massless particles of the standard model (graviton, photon, cosmon) contribute, one finds

对于给定的流方程设置和红外调节器 R_k 的选择，常数 u_0 和 u_∞ 仅依赖于有效无质量粒子 (标量、费米子、规范玻色子、引力子) 的数目。若对于 $\tilde{\rho} \to \infty$ ，只有标准模型的无质量粒子 (引力子、光子、精质子) 有贡献，可得

$$u_\infty = \frac{5}{128\pi^2} \left(-\frac{1}{128\pi^2} \right), \tag{14}$$

where the bracket also includes three generations of effectively massless neutrinos. For $\tilde{\rho} \to 0$ all effectively massless particles at the UV fixed point contribute to u_0 . One therefore expects that u_0 differs from u_∞ . The effective scalar potential $U_k(\chi)$ of the scaling solution interpolates between $u_0 k^4$ and $u_\infty k^4$. This form differs qualitatively from an almost polynomial dependence on χ that is typically found in perturbative quantum field theories for scalars. We may consider $V = u_\infty k^4$ as a type of cosmological constant induced by quantum fluctuations. Its characteristic size is given by k - the only scale present for a scaling solution.

其中方括号内还包含三代有效无质量中微子。对于 $\tilde{\rho} \to 0$ ，紫外不动点处所有有效无质量粒子都会对 u_0 产生贡献。因此可以预期 u_0 与 u_∞ 不同。标度解的有效标量势 $U_k(\chi)$ 在 $u_0 k^4$ 和 $u_\infty k^4$ 之间插值。该形式在定性上不同于微扰量子场论中标量通常得到的近似多项式依赖于 χ 的形式。我们可以将 $V = u_\infty k^4$ 视为量子涨落诱导的一类宇宙学常数。其特征大小由标度解唯一存在的标度 k 给出。

For the curvature coefficient one finds the limiting behavior

对于曲率系数，可得极限行为

$$f(\tilde{\rho} \to 0) = f_0, \ f(\tilde{\rho} \to \infty) = 2\xi_\infty \tilde{\rho}. \tag{15}$$

This implies for large χ a χ -dependent effective squared Planck mass, $F = \xi_\infty \chi^2$. The dimensionless coupling ξ_∞ is the so-called non-minimal coupling of a scalar field to gravity, according to the term $-\frac{1}{2}\xi_\infty \chi^2 R$ in the effective action. Such a coupling can already be seen in perturbation theory. There may exist additional scaling solutions with constant asymptotic value $f(\tilde{\rho} \to \infty) = f_\infty$. We focus on the generic case $\xi_\infty > 0$.

这意味着对于大 χ ，有效普朗克质量平方依赖于 χ ，即 $F = \xi_\infty \chi^2$ 。无量纲耦合 ξ_∞ 就是所谓标量场对引力的非最小耦合，对应有效作用量中的项 $-\frac{1}{2}\xi_\infty \chi^2 R$ 。这种耦合在微扰论中就已经存在。还可能存在其他具有常数渐近值 $f(\tilde{\rho} \to \infty) = f_\infty$ 的标度解。我们聚焦于通用情况 $\xi_\infty > 0$ 。

The limiting behavior (13), (15) of the scaling solutions is a central result of quantum gravity formulated as a quantum field theory with associated functional flow equations. From $u(\tilde{\rho})$ and $f(\tilde{\rho})$ one can compute the scalar effective potential in the Einstein frame $U_E(\varphi)$ (e.g., inflaton or cosmon potential) for a scalar field (see below):

标度解的极限行为 (13)、(15) 是表述为带泛函流方程的量子场论的量子引力的核心结果。由 $u(\tilde{\rho})$ 和 $f(\tilde{\rho})$ 可以计算出爱因斯坦框架下标量场的有效标量势 $U_E(\varphi)$ (例如暴胀子势或精质子势)，如下所示 (见下文):

$$U_E(\varphi) = M^4 \widehat{V}(\varphi), \ \widehat{V} = \frac{u(\tilde{\rho})}{f^2(\tilde{\rho})}, \ \varphi = 2M \ln \tilde{\rho}. \tag{16}$$

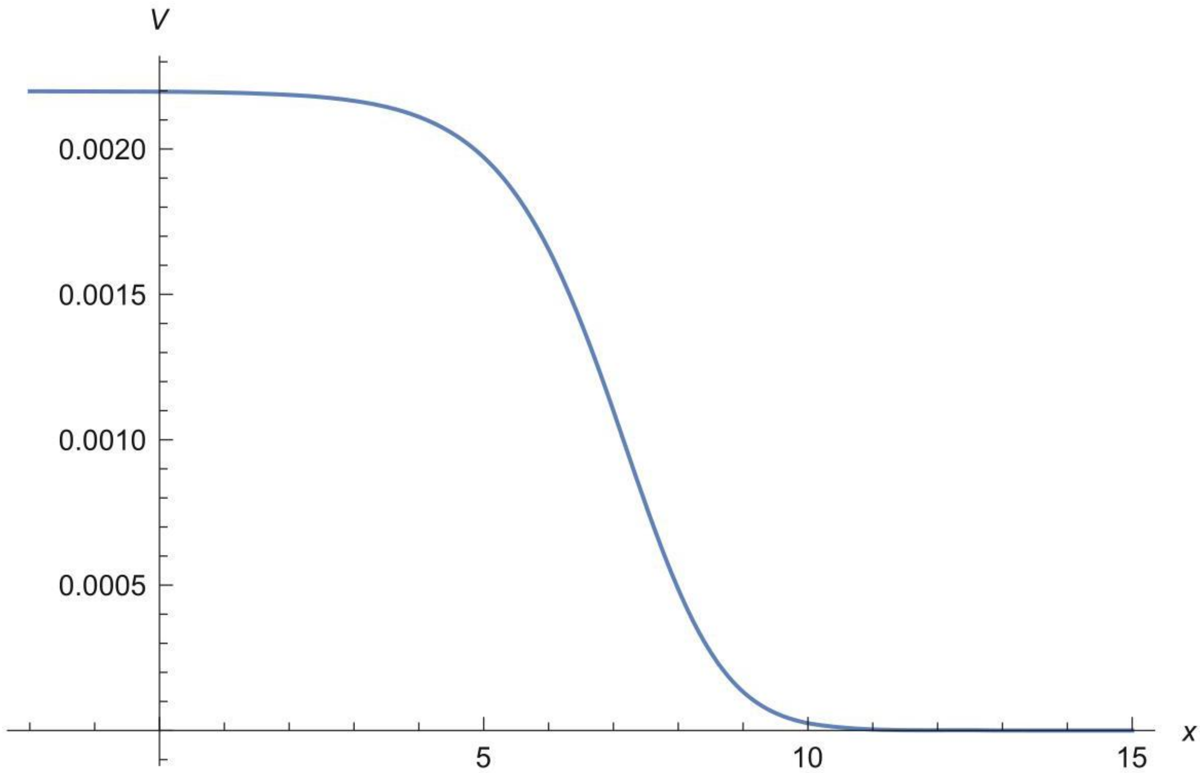

Fig. 1 Effective scalar potential. We plot $\hat{V}$, the potential in the Einstein frame in Planck units, as a function of the scalar field $x = \ln\tilde{\rho} = \varphi/(2M)$. One observes the typical flat tail for negative and small x and the exponential decrease for large x .

> 图 1 有效标量势。我们绘制了普朗克单位下爱因斯坦框架中的势 $\hat{V}$,它作为标量场 $x = \ln\tilde{\rho} = \varphi/(2M)$ 的函数。可以观察到，当负的和小的 x 时出现典型的平尾，大 x 时呈指数衰减。

We plot this potential in Fig. 1. It exhibits a flat tail suitable for inflation for $\varphi \to -\infty$ and an exponential decrease characteristic for some form of dynamical dark energy for increasing φ . The effective potential vanishes for $\varphi \to \infty$.

> 我们在图 1 中绘制了该势。对于 $\varphi \to -\infty$ ，它呈现出适合暴胀的平尾；随着 φ 增大，它呈现出一类动力学暗能量特有的指数衰减特征。有效势在 $\varphi \to \infty$ 处为零。

We will argue below that this behavior is generic and rather robust. It has important consequences for cosmology. For the kinetial $K(\tilde{\rho})$ no similarly robust results are available at present in our view, despite several encouraging first computations [65].

> 我们下文将指出，该行为具有普适性，稳定性很强。它对宇宙学有重要影响。我们认为，尽管已有多个令人鼓舞的初步计算 [65]，目前对于动力学项 $K(\tilde{\rho})$ 还没有同样可靠的结果。

Relevant Parameters and Predictivity of Quantum Gravity

量子引力的相关参数与可预测性

For a complete quantum field theory of gravity, it is sufficient that the scaling solution is approached for $k \to \infty$ or $\tilde{\rho} \to 0$. Starting with the arbitrarily large finite k in the arbitrarily close vicinity of the scaling solution, the flow of Γ_k towards lower k may deviate from the scaling solution. This flow away from the scaling solution is typically determined in terms of a small number of "relevant parameters." To every relevant parameter one can associate a free parameter or renormalized coupling in the macroscopic quantum effective action $\Gamma_{k\to 0}$. Further, free parameters can arise if the scaling solution is not unique. A family of scaling solutions may be specified by continuous parameters. In any case the number of free parameters in the quantum effective action is finite. This renders our approach to quantum gravity rather predictive. If the number of free parameters is smaller than the number of renormalizable couplings in the standard model of particle physics, relations between the latter can be predicted. For cosmology one expects important restrictions on the functions U, F , and K , such that models with a given particle content can be tested by cosmological observations.

对于完整的引力量子场论，只要标度解对 $k \to \infty$ 或 $\tilde{\rho} \to 0$ 逼近就足够了。若任意大的有限 k 起始于标度解任意近的邻域内，Γ_k 向更低 k 的流可能会偏离标度解。这种偏离标度解的流动通常由少量“相关参数”决定。每个相关参数都对应宏观量子有效作用量 $\Gamma_{k\to 0}$ 中的一个自由参数或可重整化耦合。此外，如果标度解不唯一，也会产生自由参数。一族标度解可以由连续参数描述。无论如何，量子有效作用量中的自由参数数量是有限的。这使得我们的量子引力方法具有相当强的预测性。如果自由参数的数量小于粒子物理标准模型中可重整化耦合的数量，就可以预言后者之间的关联。对于宇宙学，我们预期会对函数 U, F 和 K 给出重要限制，因此具有给定粒子内容的模型可以通过宇宙学观测检验。

One of the free parameters sets the overall mass or momentum scale. Specifying only units, this is not an observable parameter. We may define a characteristic scale k_c for which the solution of the flow equation starts to deviate substantially from the scaling solution due to the presence of relevant parameters. For $k > k_c$ one therefore can use the scaling solution as a good approximation. We can associate arbitrary energy units to k_c . Only dimensionless quantities as χ/k_c will be observable. We will discuss in section "Fundamental Scale Invariance" the attractive possibility of fundamental scale invariance [146] for which the scaling solution remains exact for all k . All relevant parameters vanish in this case, and the predictivity of the model is enhanced further. In this case the energy units are determined by k , which can again be chosen freely.

其中一个自由参数设定了整体质量或动量标度。它仅用于指定单位，并不是可观测参数。我们可以定义一个特征标度 k_c ，由于相关参数的存在，流方程的解在该标度处开始显著偏离标度解。因此对于 $k > k_c$ ，可以将标度解作为良好近似。我们可以为 k_c 赋予任意能量单位。只有像 χ/k_c 这样的无量纲量才是可观测的。我们将在“基本标度不变性”一节中讨论基本标度不变性 [146] 这个引人注目的可能性: 在该理论中标度解对所有 k 都保持精确。这种情况下所有相关参数都为零，模型的可预测性进一步提升。此时能量单位由 k 确定，而 k 仍可自由选择。

Quantum Scale-Invariant Standard Model

量子标度不变标准模型

In this note we mainly concentrate on the extended gravitational sector of the metric and the cosmon. These fields couple to other particles, as fermions, gauge bosons, and additional scalars. The scaling solution requires that for $k \to 0$ all particle physics mass scales as the Fermi scale (expectation value of the Higgs scalar) or the confinement scale in quantum chromodynamics are proportional to χ . For arbitrary k the electron mass takes the form $m_e = h_e(\tilde{\rho})\chi$ and similarly the nucleon mass $m_N = h_N(\tilde{\rho})\chi$. In the limit $k \to 0$ or $\tilde{\rho} \to \infty$, the effective dimensionless coupling functions approach constants $h_{e,\infty}$ and $h_{N,\infty}$. As a consequence, the observable mass ratios, electron mass over Planck mass or electron mass over nucleon mass,

在本文中，我们主要关注度规和宇宙子的扩展引力部分。这些场与其他粒子耦合，例如费米子、规范玻色子和额外标量场。标度解要求，对于 $k \to 0$, 所有粒子物理质量标度——即希格斯标量的期望值——或者量子色动力学中的禁闭标度都与 χ 成正比。对于任意 k , 电子质量形式为 $m_e = h_e(\tilde{\rho})\chi$, 核子质量同理为 $m_N = h_N(\tilde{\rho})\chi$ 。在极限 $k \to 0$ 或 $\tilde{\rho} \to \infty$ 下，有效无量纲耦合函数趋近常数 $h_{e,\infty}$ 和 $h_{N,\infty}$ 。因此，可观测质量比，即电子质量与普朗克质量之比、电子质量与核子质量之比，

$$\frac{m_e}{\sqrt{F}} = \frac{h_{e,\infty}}{\sqrt{\xi_\infty}}, \ \frac{m_e}{m_N} = \frac{h_{e,\infty}}{h_{N,\infty}}, \tag{17}$$

approach constants. Furthermore, dimensionless renormalizable couplings as the fine structure constant or Yukawa couplings of quarks and leptons approach constants for $\tilde{\rho} \to \infty$. For $k \to 0$ these couplings do not depend on χ . Even though χ typically changes in the course of the cosmological evolution one finds no time dependent fundamental couplings [19, 22, 26, 27, 29, 39, 85, 122, 127, 129, 152] for the scaling solution in this limit. Also apparent violations of the equivalence principle by a scalar-mediated fifth force are absent. The matter-dominated universe shows the same observable features as for Einstein gravity, in contrast to cosmologies with a varying Planck mass and fixed particle masses [8, 49, 126] .

趋近于常数。此外，对于 $\tilde{\rho} \to \infty$, 精细结构常数、夸克与轻子汤川耦合这类无量纲可重整耦合都会趋近常数。对于 $k \to 0$, 这些耦合不依赖于 χ 。即使 χ 通常会随宇宙演化过程发生变化，在该极限下的标度解中，也不存在随时间变化的基本耦合 [19, 22, 26, 27, 29, 39, 85, 122, 127, 129, 152] 。标量介导的第五力也不会造成明显的等效原理破缺。与普朗克质量可变、粒子质量固定的宇宙学 [8, 49, 126] 不同，物质主导宇宙具有和爱因斯坦引力完全相同的可观测特征。

For low enough momenta (below the effective Planck mass or some grand unification scale), the scaling solution in the limit $k \to 0$ amounts to the quantum scale-invariant standard model [112-114, 128]. No intrinsic mass scale is present in the quantum effective action. All particle masses, cross sections, etc. are proportional to appropriate powers of χ according to their dimension. The quantum scale-invariant standard model is the basis for discussions of dark energy [10, 73 , 128] and different versions of scale-invariant inflation [15, 16, 45, 46, 53, 107].

对于足够低的动量 (低于有效普朗克质量或某种大统一标度)，极限 $k \to 0$ 下的标度解就是量子标度不变标准模型 [112-114, 128]。量子有效作用量中不存在内禀质量标度。所有粒子质量、截面等，根据其量纲都与 χ 的对应幂次成正比。量子标度不变标准模型是讨论暗能量 [10, 73 , 128] 和不同版本标度不变暴胀 [15, 16, 45, 46, 53, 107] 的基础。

The quantum scale-invariant standard model remains a very good approximation for the range of χ or $\tilde{\rho}$ for which k_c or k are much smaller than all mass scales of the standard model. We will consider models of this type, setting k_c or k in the order 10^{-2}eV , which is many orders of magnitude smaller than the electron mass. As a consequence, the radiation and matter-dominated epochs in cosmology will be given by Einstein gravity, with possible small modifications due to the presence of the scalar field χ , which may account for a small fraction of early dark energy [36, 61, 131].

对于 χ 或 $\tilde{\rho}$ 满足 k_c 或 k 远小于标准模型所有质量标度的范围，量子标度不变标准模型依然是非常好的近似。我们将讨论这类模型，取 k_c 或 k 量级为 10^{-2}eV，这比电子质量小很多个数量级。因此，宇宙学中的辐射主导纪元和物质主导纪元都符合爱因斯坦引力，标量场 χ 的存在只会带来可能的微小修正，该修正可以解释早期暗能量的很小占比 [36, 61, 131]。

The quantum scale-invariant standard model does not imply that there are no running couplings. We have focused so far on vanishing momenta, as appropriate for cosmology. For scattering processes at nonzero squared momenta q^2 couplings as the fine structure constant α are running couplings. According to the scaling solution they depend on the dimensionless ratios $\tilde{\rho}$ and q^2/χ^2 . Quantum effects induce a running of the couplings with q^2 at fixed χ^2 , corresponding to fixed particle masses. This running follows the perturbative β -functions, with appropriate mass thresholds for the decoupling of particles [89].

量子标度不变标准模型并不意味着不存在跑动耦合。到目前为止，我们聚焦于适用于宇宙学的零动量情形。对于非零平方动量 q^2 下的散射过程，精细结构常数 α 这类耦合是跑动耦合。根据标度解，它们依赖于无量纲比 $\tilde{\rho}$ 和 q^2/χ^2 。在固定 χ^2 对应固定粒子质量的情况下，量子效应会引发耦合随 q^2 跑动，这种跑动遵循微扰 β 函数，且带有粒子退耦对应的合适质量阈值 [89]。

Cosmology for the Scaling Solution

标度解的宇宙学

For our discussion of cosmology we concentrate on the functions U, F , and K according to the scaling solution. This covers both the setting of fundamental scale invariance and the case where the deviation from the scaling solution occurs at a scale k_c . In the latter case we assume that the flow stops for $k \ll k_c$. In this approximation we can use the scaling solution with k identified with k_c . Corrections to this simplified treatment involve for U or F a dependence on k/k_c which describes the deviation from the scaling solution. For $k \gg k_c$ this deviation vanishes. The effect of this ”final running” on the difference between $U(k=0)$ and $U(k=k_c)$ is typically a small constant $\sim k_c^4$ and similar for F with a constant $\sim k_c^2$. We will discuss this point in section ”Fundamental Scale Invariance.”

在本文的宇宙学讨论中，我们根据标度解重点研究函数 U, F 和 K 。该讨论既涵盖基本标度不变性的设定，也涵盖偏离标度解发生在标度 k_c 处的情况。对于后一种情况，我们假设当 $k \ll k_c$ 时流停止。在此近似下，我们可使用标度解，其中 k 对应 k_c 。对该简化处理的修正中，对于 U 或 F ，其依赖于描述对标度解偏离的 k/k_c 。对于 $k \gg k_c$ ，该偏离消失。这种“最终跑动”对 $U(k=0)$ 和 $U(k=k_c)$ 之差的影响通常是一个小常数 $\sim k_c^4$ ，类似地，F 对应一个常数 $\sim k_c^2$ 。我们将在“基本标度不变性”一节讨论这一点。

Inserting the limiting behavior of the scaling solution in the frame-invariant dimensionless potential $\hat{V}$ (7) yields

将标度解的极限行为代入规范不变无量纲势 $\hat{V}$ (7)，可得

$$\hat{V} = \frac{u}{f^2},\ \hat{V}(\chi \to 0) = \frac{u_0}{f_0^2},\ \hat{V}(\chi \to \infty) = \frac{u_\infty k^4}{\xi_\infty^2 \chi^4}. \tag{18}$$

This potential approaches a constant for $\chi \to 0$ and vanishes $\sim \chi^{-4}$ for $\chi \to \infty$. We will find solutions of the cosmological field equations (5),(6) for which χ evolves from zero for $\eta \to -\infty$ to infinity for $\eta \to \infty$. The regime of small χ^2 will be associated to inflation, while the region of large χ^2 will account for dynamical dark energy. The exact vanishing of $\hat{V}$ for $\chi \to \infty$ is associated to a (dynamical) solution of the cosmological constant problem.

当 $\chi \to 0$ 时，该势趋近于常数；当 $\chi \to \infty$ 时，该势变为零 $\sim \chi^{-4}$ 。我们将得到宇宙场方程 (5)、(6) 的解: 对于这类解，χ 从 $\eta \to -\infty$ 时的零演化到 $\eta \to \infty$ 时的无穷大。小 χ^2 区域对应暴胀，大 χ^2 区域对应动力学暗能量。当 $\chi \to \infty$ 时 $\hat{V}$ 恰好为零，这对应宇宙学常数问题的一个 (动力学) 解。

Quantum Scale Symmetry

量子标度对称性

Quantum scale symmetry [143] is a key concept for the understanding of dominant features of cosmology beyond Einstein gravity. It is directly related to the scaling solution of the flow equation and associated fixed points. At fixed points quantum scale symmetry becomes exact. For early cosmology the observed approximate scale invariance of the spectrum of primordial cosmic fluctuations can find its root in the approximate quantum scale symmetry for the vicinity of an ultraviolet (UV) fixed point for $k \to \infty$ or $\chi \to 0$. Late cosmology describes the approach to an infrared (IR) fixed point for $k \to 0$ or $\chi \to \infty$. Precisely at the infrared fixed point quantum scale symmetry will be an exact global symmetry of the quantum effective action. It is, however, spontaneously broken by the nonzero value of χ . Any spontaneously broken exact global symmetry predicts the presence of a massless Goldstone boson - the dilaton in our case. For finite large $\tilde{\rho}$ quantum scale symmetry is only approximate, resulting in a tiny mass for the pseudo-Goldstone boson which is associated to the cosmon. Spontaneously broken approximate quantum scale symmetry gives therefore a natural reason for a very light scalar field which can provide for dynamical dark energy [128].

量子标度对称性 [143] 是理解超越爱因斯坦引力的宇宙学核心特征的关键概念。它直接与流方程的标度解及相关不动点关联。在不动点处，量子标度对称性变得精确。对于早期宇宙学，观测到的原初宇宙涨落谱的近似标度不变性，其根源可追溯到 $k \to \infty$ 或 $\chi \to 0$ 在紫外 (UV) 不动点附近的近似量子标度对称性。晚期宇宙学描述了向 $k \to 0$ 或 $\chi \to \infty$ 的红外 (IR) 不动点趋近的过程。在红外不动点处，量子标度对称性恰好是量子有效作用量的精确整体对称性。然而它会因 χ 的非零值发生自发破缺。任何自发破缺的精确整体对称性都预言存在无质量哥德斯通玻色子——在本文中即 dilation(dilaton，dilation 子)。对于有限大的 $\tilde{\rho}$ ，量子标度对称性仅为近似，这使得与宇宙场 (cosmon) 关联的赝哥德斯通玻色子获得极小质量。因此，自发破缺的近似量子标度合理解释了极轻标量场的存在，而该场可以提供动力学暗能量 [128]。

Quantum scale symmetry emerges as an exact global symmetry whenever the quantum effective action Γ does not involve any intrinsic momentum or length scale. All scales are then given by fields as χ . For the scaling solution of the flow equation, this global symmetry is realized if for a suitable choice of fields the effective action becomes independent of k for fixed fields. In our setting this typically occurs for $\tilde{\rho} \to 0$ (UV fixed point) and for $\tilde{\rho} \to \infty$ (IR fixed point). More precisely, quantum scale symmetry is an exact global symmetry at fixed points where all scale dependence can be absorbed in renormalized fields which transform nontrivially under scale transformations. While classical scale symmetry is broken by quantum effects leading to running dimensionless couplings, quantum scale symmetry is generated by the quantum fluctuations. It is the flow of the couplings and the associated fixed points that is responsible for this symmetry.

当量子有效作用量 Γ 不包含任何内禀动量或长度标度时，量子标度对称性就会作为精确整体对称性出现。此时所有标度都由场给出，如 χ 。对于流方程的标度解，若选取合适的场，有效作用量在固定场下与 k 无关，该整体对称性就得以实现。在我们的框架中，这通常发生在 $\tilde{\rho} \to 0$ (紫外不动点) 和 $\tilde{\rho} \to \infty$ (红外不动点) 处。更准确地说，在不动点处量子标度对称性是精确整体对称性，此时所有标度依赖性都可以被重整化场吸收，这些场在标度变换下发生非平凡变换。经典标度对称性会被量子效应破缺，导致无量纲耦合跑动，而量子标度对称性由量子涨落产生，正是耦合的流动和相关不动点催生了该对称性。

Infrared Fixed Point

红外不动点

For $\tilde{\rho} \to \infty$ the quantum effective action takes the simple form

对于 $\tilde{\rho} \to \infty$ ，量子有效作用量取简单形式

$$\Gamma = \int_x \sqrt{g}\left\{-\frac{1}{2}\xi_\infty \chi^2 R + u_\infty k^4 + \frac{1}{2}K\left(\frac{\chi^2}{k^2}\right)\partial^\mu \chi \partial_\mu \chi\right\}. \tag{19}$$

By rescaling χ we can set $\xi_\infty \to 1$, with new kinetial $\widetilde{K} = K/\xi_\infty$. The quantum scale transformations or dilatations act as

通过重标度化 χ ，我们可以令 $\xi_\infty \to 1$ ，得到新动能项 $\widetilde{K} = K/\xi_\infty$ 。量子标度变换或标度变换的作用形式为

$$g_{\mu\nu} \to \alpha^{-2} g_{\mu\nu},\ \chi \to \alpha\chi \tag{20}$$

For $\tilde{\rho} \to \infty$ the constant potential $u_\infty k^4$ can be neglected, and quantum scale symmetry becomes exact if $\widetilde{K}$ approaches a constant. The violations of scale symmetry close to the IR fixed point arise for nonzero k from $u_\infty k^4$ and a possible $\tilde{\rho}$ -dependence of $\widetilde{K}$. Alternatively, by use of the freedom of an overall rescaling of χ , we may set $K_\infty = K(\chi \to \infty) = \pm 1$. With this normalization quantum scale symmetry is realized if $\xi(\chi \to \infty)$ takes a constant value ξ_∞ . We will see that a negative value K_∞ can be consistent with stability.

对于 $\tilde{\rho} \to \infty$ ，常数势 $u_\infty k^4$ 可以忽略，若 $\widetilde{K}$ 趋近于常数，则量子标度对称性变得精确。当接近红外不动点时，非零的 k 会因 $u_\infty k^4$ 和 $\widetilde{K}$ 可能存在的 $\tilde{\rho}$ 依赖产生标度对称性破缺。或者，利用 χ 整体重标度化的自由度，我们可以令 $K_\infty = K(\chi \to \infty) = \pm 1$ 。在此归一化下，当 $\xi(\chi \to \infty)$ 取常数 ξ_∞ 时，量子标度对称性得以实现。我们会看到，负值 K_∞ 可以满足稳定性要求。

As mentioned above, the scaling solution implies that the dimensionless potential $\widehat{V}$ vanishes for $\chi \to \infty$. Quantum scale symmetry alone does not guarantee this behavior, since a scale-invariant potential $U = \lambda\chi^4$ would lead to constant $\widehat{V}$. The vanishing of λ is required by the scaling solution which only exists if $u(\tilde{\rho} \to \infty)$ approaches a constant. On rather general grounds, one can show [140] that in the presence of metric fluctuations, the potential U cannot grow faster than F for $\chi \to \infty$. This rule is obeyed for an asymptotically constant U , but not for $U \sim \chi^4$.

如上所述，标度解意味着无量纲势 $\widehat{V}$ 在 $\chi \to \infty$ 处为零。仅靠量子标度对称性无法保证这一行为，因为标度不变势 $U = \lambda\chi^4$ 会给出常数 $\widehat{V}$ 。λ 为零是标度解的要求，标度解仅当 $u(\tilde{\rho} \to \infty)$ 趋近于常数时存在。基于相当一般的前提，可以证明 [140]，当存在度规涨落时，在 $\chi \to \infty$ 处势 U 的增长速度不会超过 F 。渐近常数的 U 满足这一规则，但 $U \sim \chi^4$ 不满足。

Weyl Scaling

外尔标度

The physical implications of the effective action (19) are most easily understood by performing a field transformation of the metric. By the conformal transformation or Weyl scaling [31, 155],

通过对度规进行场变换，可以最简便地理解有效作用量 (19) 的物理含义。通过共形变换即外尔标度 [31, 155],

$$g'_{\mu\nu} = w^{-2}(\chi)\, g_{\mu\nu}, \tag{21}$$

the effective action (3) retains its form when expressed in terms of the new metric $g'_{\mu\nu}$. The transformed functions are

用新度规 $g'_{\mu\nu}$ 表示时，有效作用量 (3) 保持形式不变，变换后的函数为

$$F' = w^2 F,\ U' = w^4 U,$$

$$K' = w^2 \left[K - 6F \frac{\partial \ln w}{\partial \chi} \left(\frac{\partial \ln w}{\partial \chi} + \frac{\partial \ln F}{\partial \chi} \right) \right]. \tag{22}$$

The frame-invariant combinations $\widehat{V}$ and $\widehat{K}$ in Eq. (7) remain the same when expressed in terms of U', F', and K'. The field equations (5),(6) hold for all metric frames related by an arbitrary choice of $w(\chi)$. Both conformal time η and the combination A are invariant under Weyl scalings. For many observables the independence from the choice of the metric frame has been demonstrated explicitly [18,20,25,30,44,69,70,72,100,106,135,148]. Weyl scalings change the geometry without affecting observables. Different metric frames often induce unusual pictures of cosmology [134, 147].

式 (7) 中的洛伦兹不变组合 $\widehat{V}$ 和 $\widehat{K}$ 用 U', F' 和 K' 表示时保持不变。场方程 (5)、(6) 对所有由任意 $w(\chi)$ 选择关联的度规框架都成立。共形时间 η 和组合 A 在外尔标度下都是不变量。对于很多可观测物理量, 度规框架选择的无关性已经得到了明确证明 [18,20,25,30,44,69,70,72,100,106,135,148]。外尔标度改变几何, 但不影响可观测物理量。不同的度规框架往往会给宇宙学带来非常规的描述 [134, 147]。

The Einstein frame obtains for a choice $w^2 = M^2/F$, such that the curvature coefficient is given by the squared Planck mass, $F' = M^2$. We emphasize that the Planck mass M is introduced only by the variable transformation (21), rather than being a parameter of the model. The scalar potential in the Einstein frame U_E is directly related to the frame-invariant potential $\widehat{V}$ by $U_E = M^4 \widehat{V}$. For the vicinity of the IR fixed point with $F = \xi_\infty \chi^2$, one finds for the kinetial in the Einstein frame

当选择 $w^2 = M^2/F$ 使得曲率系数由普朗克质量平方 $F' = M^2$ 给出时，就得到爱因斯坦框架。我们强调，普朗克质量 M 仅由变量变换 (21) 引入，并非模型本身的参数。爱因斯坦框架下的标量势 U_E 与洛伦兹不变势 $\widehat{V}$ 通过 $U_E = M^4 \widehat{V}$ 直接关联。对于满足 $F = \xi_\infty \chi^2$ 的红外固定点邻域，可得爱因斯坦框架下的动力学项为

$$K_E = \frac{M^2}{\chi^2} \left(\frac{K}{\xi_\infty} + 6 \right). \tag{23}$$

The factor χ^{-2} can be absorbed by defining

因子 χ^{-2} 可以通过定义吸收,

$$\varphi = 4M \ln \left(\frac{\chi}{k} \right), \tag{24}$$

such that the effective action (19) reads in the Einstein frame,

因此有效作用量 (19) 在爱因斯坦框架下写为

$$\Gamma_E = \int_x \sqrt{g_E} \left\{ -\frac{1}{2} M^2 R_E + U_E(\varphi) + \frac{1}{2} Z(\varphi) \partial^\mu \varphi \partial_\mu \varphi \right\}, \tag{25}$$

with potential

其势能为

$$U_E(\varphi) = \frac{u_\infty M^4}{\xi_\infty^2} \exp\left(-\frac{\varphi}{M}\right) \tag{26}$$

and kinetial

动力学项为

$$Z(\varphi) = \frac{1}{16}\left(\frac{K}{\xi_\infty} + 6\right). \tag{27}$$

We observe that the criterion of stability is $Z(\varphi) \geq 0$, such that K can actually be negative provided $K > -6\xi_\infty$.

我们发现稳定性判据为 $Z(\varphi) \geq 0$ ，因此只要满足 $K > -6\xi_\infty$ ，K 实际可以为负值。

The potential in the Einstein frame vanishes exponentially for $\varphi \to \infty$, corresponding to the vanishing of $\hat{V}$ for $\chi \to \infty$. Exponential potentials are often used for models of quintessence [24, 47, 92, 123, 128] . We could use a further field transform of the scalar field to bring the kinetic term to a canonical form. We will not always do so since the discussion of dynamical dark energy with a field-dependent kinetial is actually quite convenient [52, 63, 71, 130, 136, 138] . The general form of Z is given by

对于 $\varphi \to \infty$ ，爱因斯坦框架下的势能指数衰减为零，对应 $\chi \to \infty$ 处 $\hat{V}$ 为零。指数势能常被用于精质模型 [24, 47, 92, 123, 128] 。我们可以对标量场进一步做场变换，将动能项化为标准形式。我们通常不这么做，因为讨论带场依赖动力学项的动力学暗能量实际上非常方便 [52, 63, 71, 130, 136, 138] 。Z 的一般形式为

$$Z = \frac{K\tilde{\rho}}{8f} + \frac{3}{8}\left(\frac{\partial \ln f}{\partial \ln \tilde{\rho}}\right)^2, \; 2\tilde{\rho} = \exp\left(\frac{\varphi}{2M}\right). \tag{28}$$

Stability requires $Z(\tilde{\rho}) \geq 0$ for all $\tilde{\rho}$.

稳定性要求对所有 $\tilde{\rho}$ 都满足 $Z(\tilde{\rho}) \geq 0$ 。

Cosmon as Pseudo-Goldstone Boson of Quantum Scale Symmetry

哥斯蒙子作为量子标度对称性的赝戈德斯通玻色子

The scalar field σ with canonical kinetic term is related to φ by

具有正则动能项的标量场 σ 通过下式与 φ 关联

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\varphi} = Z^{1/2}(\varphi) \tag{29}$$

With this normalization the mass m_c of the cosmon obeys

在该归一化下，哥斯蒙子的质量 m_c 满足

$$m_c^2 = \frac{\partial^2 U_E}{\partial \sigma^2} = \left(1 + \frac{M}{2}\frac{\partial \ln Z}{\partial \varphi}\right)\frac{U_E}{ZM^2}$$

$$= \left[1 + \frac{1}{4}\frac{\partial}{\partial \ln \tilde{\rho}}\ln\left(\frac{K}{\xi_\infty} + 6\right)\right]\frac{U_E}{ZM^2}. \quad (30)$$

For $\varphi \to \infty$ or $\tilde{\rho} \to \infty$ the ratio K/ξ_∞ becomes independent of $\tilde{\rho}$ if a fixed point is reached. The mass of the cosmon vanishes if this limit is proportional to U_E/Z , as expected for a pseudo-Goldstone boson with explicit symmetry breaking given by U_E .

若达到固定点，对于 $\varphi \to \infty$ 或 $\tilde{\rho} \to \infty$ ，比值 K/ξ_∞ 与 $\tilde{\rho}$ 无关。若该极限与 U_E/Z 成正比，哥斯蒙子质量将消失，这符合由 U_E 给出显对称性破缺的赝戈德斯通玻色子的预期。

For $(K/\xi_\infty)(\tilde{\rho} \to \infty) = -6$, the global scale symmetry at the IR fixed point is enhanced to a local "Weyl symmetry" with spacetime-dependent parameter $\alpha(x)$ in Eq. (20). This includes conformal symmetry. In this limit the scalar field ceases to be a propagating degree of freedom, as seen directly from $Z(\varphi \to \infty) = 0$. Indeed, the metric in the Einstein frame $g'_{\mu\nu} = \left(\xi_\infty \chi^2/M^2\right) g_{\mu\nu}$ is invariant under local Weyl scalings. The local Weyl scaling acts as $\varphi(x) \to \varphi(x) + 4M \ln \alpha(x)$. Local Weyl symmetry is realized if the effective action in the Einstein frame does not involve the scalar field φ . If the IR cutoff respects local Weyl symmetry (cf. Refs. [23,95]), this enhanced symmetry is a partial fixed point of the flow equations. (This always holds if the flow equations are compatible with the enhanced symmetry.) If this partial fixed point plays a role for the scaling solution for $\tilde{\rho} \to \infty$, one expects that $Z(\varphi)$ vanishes for $\varphi \to \infty$. For Z vanishing slower than exponentially, the cosmon mass still approaches zero for $\varphi \to \infty$.

对于 $(K/\xi_\infty)(\tilde{\rho} \to \infty) = -6$ ，红外固定点处的整体标度对称性增强为局域“外尔对称性”，其依赖时空的参数为式 (20) 中的 $\alpha(x)$ ，该对称性包含共形对称性。在此极限下，标量场不再是传播自由度，这可从 $Z(\varphi \to \infty) = 0$ 直接看出。事实上，爱因斯坦度规 $g'_{\mu\nu} = \left(\xi_\infty \chi^2/M^2\right) g_{\mu\nu}$ 在局域外尔标度变换下保持不变。局域外尔标度变换的作用为 $\varphi(x) \to \varphi(x) + 4M \ln \alpha(x)$ 。若爱因斯坦框架下的有效作用量不包含标量场 φ ，则局域外尔对称性得以实现。若红外截断满足局域外尔对称性 (参见文献 [23,95])，该增强对称性就是流方程的部分固定点。(当流方程与增强对称性相容时该结论始终成立。) 若该部分固定点对 $\tilde{\rho} \to \infty$ 的标度解有影响，可预期对于 $\varphi \to \infty$ ，$Z(\varphi)$ 趋近于零。若 Z 慢于指数衰减，哥斯蒙子质量在 $\varphi \to \infty$ 下仍会趋近于零。

A Weyl transformation to the Einstein frame has also to be applied to fermions and other scalars in order to ensure that a standard normalization of the kinetic terms remains preserved. At the fixed point for $\varphi \to \infty$, the dimensionless mass ratios or couplings become independent of φ in the Einstein frame. As a result, the cosmon can only have derivative couplings, as appropriate for a Goldstone boson. We observe that in the Einstein frame the (global) scale transformation (20) acts as a constant shift

为保证动能项始终保持标准归一化，变换到爱因斯坦框架的外尔变换也需要应用于费米子和其他标量场。在 $\varphi \to \infty$ 的固定点处，无量纲质量比或耦合在爱因斯坦框架下与 φ 无关。因此，哥斯蒙子只能具有导数耦合，这符合戈德斯通玻色子的特性。我们可以看到，爱因斯坦框架下 (整体) 标度变换 (20) 表现为一个常数平移

$$\varphi \to \varphi + 4M \ln \alpha \quad (31)$$

while the metric $g_{E\mu\nu} = g_{\mu\nu}\xi_\infty \chi^2/M^2$ as well as the rescaled fields for fermions, other scalars, and gauge bosons are invariant. This shift symmetry implies directly the absence of nonderivative couplings of φ .

而度规 $g_{E\mu\nu} = g_{\mu\nu}\xi_\infty \chi^2/M^2$ 以及费米子、其他标量、规范玻色子的重标度场保持不变。该平移对称性直接说明 φ 不存在非导数耦合。

Dynamical Dark Energy

动力学暗能量

The potential and kinetic terms of the cosmon φ are a source of dynamical dark energy, according to the field equation

根据场方程，宇称场 φ 的势项和动能项是动力学暗能量的一个源

$$H^2 = \frac{1}{3M^2}\left[U_E + \frac{Z}{2}(\partial_t\varphi)^2 + \rho_E\right], \tag{32}$$

where we have added to Eq. (5) the contribution of the energy density in radiation and matter, given by ρ_E in the Einstein frame. The scalar field evolves according to Eq. (6)

其中我们在式 (5) 中加入了辐射和物质的能量密度贡献，由爱因斯坦框架下的 ρ_E 给出。标量场根据式 (6) 演化

$$\left(\partial_t^2 + 3H\partial_t\right)\varphi + \frac{1}{2}\frac{\partial \ln Z}{\partial\varphi}(\partial_t\varphi)^2 = -\frac{1}{Z}\frac{\partial U_E}{\partial\varphi}$$

$$= \frac{U_E}{MZ} = \frac{M^3}{Z}\exp\left[-\frac{\varphi}{M} + \ln\left(\frac{u_\infty}{\xi_\infty^2}\right)\right].$$

(33)

In the limit where the term $\sim \partial \ln Z/\partial\varphi$ can be neglected, the scalar field φ "rolls down" an exponential potential, increasing to infinity as cosmic time t (or equivalently conformal time η) goes to infinity. Thus, the infrared fixed point at $\varphi \to \infty$ is approached asymptotically in the infinite future of the cosmic evolution. The parameters u_∞, ξ_∞ can be absorbed by a constant shift of φ .

在项 $\sim \partial \ln Z/\partial\varphi$ 可以忽略的极限下，标量场 φ 沿指数势“滚下”，随宇宙时间 t (或等价地共形时间 η) 趋于无穷而增长至无穷。因此，宇宙演化在未来无限远处渐近趋近 $\varphi \to \infty$ 处的红外不动点。参数 u_∞, ξ_∞ 可以通过 φ 的常数平移吸收。

The homogeneous dark energy density ρ_h is given by

均匀暗能量密度 ρ_h 由下式给出

$$\rho_h = U_E + \frac{Z}{2}(\partial_t\varphi)^2 = U_E + T_E, \tag{34}$$

and the equation of state w_h is defined by

物态方程 w_h 定义为

$$w_h = \frac{T_E - U_E}{T_E + U_E},\ T_E = \frac{1}{2}\left(1 + w_h\right)\rho_h. \tag{35}$$

Multiplying Eq. (32) with $Z\partial_t\varphi$ yields the ”conservation equation”

将式 (32) 乘以 $Z\partial_t\varphi$ 得到“守恒方程”

$$\partial_t\rho_h + 6HT_E = \partial_t\rho_h + 3H\left(1 + w_h\right)\rho_h = 0. \tag{36}$$

This may be compared with the conservation equation for ρ_E ,

这可以和 ρ_E 的守恒方程对比,

$$\partial_t\rho_E = nH\rho_E, \tag{37}$$

with $n = 3$ for matter domination and $n = 4$ for radiation domination. For $w_h > 0$ dark energy decreases faster than matter, while for $w_h < 0$ the decrease of ρ_h is slower than matter such that the energy density in the scalar field may finally dominate. For the matter-dominated universe there exists a possible ”cosmic scaling solution” if $w_h = 0$. In this case dark energy decreases at the same rate as matter, such that the fraction of dark energy,

物质主导时为 $n = 3$ ，辐射主导时为 $n = 4$ 。当 $w_h > 0$ 时暗能量比物质衰减更快，而当 $w_h < 0$ 时 ρ_h 的衰减比物质更慢，因此标量场的能量密度最终会占据主导。对于物质主导宇宙，若满足 $w_h = 0$ 则存在可能的“宇宙标度解”。这种情况下暗能量和物质衰减速率相同，因此暗能量占比

$$\Omega_h = \frac{\rho_h}{\rho_h + \rho_E} = \frac{\rho_h}{3M^2H^2}, \tag{38}$$

becomes a constant. For the radiation-dominated universe a cosmic scaling solution with constant Ω_h is realized for $w_h = 1/3$. With

成为常数。对于辐射主导宇宙，当 $w_h = 1/3$ 时会实现 Ω_h 为常数的宇宙标度解。结合

$$y = \ln\left(aM\right) = \ln\left(A\right), \tag{39}$$

we can combine the conservation equations to

我们可以将多个守恒方程合并为

$$\partial_y\Omega_h = -\left[3\left(1 + w_h\right) + 2\partial_y \ln H\right]\Omega_h$$

$$= [n - 3(1 + w_h)](1 - \Omega_h) \,. \tag{40}$$

The last equation holds for all metric frames if we use $y = \ln A$.

若使用 $y = \ln A$ ，最后一个方程对所有度规框架都成立。

The detailed dynamics of dark energy requires knowledge about the φ -dependence of Z . We will discuss this in section "Crossover Cosmology." For constant $Z < 1/n$ we will indeed find cosmic scaling solutions with constant $\Omega_h = Zn$ [128]. They are attractors in the sense that neighboring solutions approach for increasing time these cosmic scaling solutions. Cosmic scaling solutions can give a natural explanation why the present value $\rho_h/M^4 \approx 10^{-120}$ is so tiny. With constant Ω_h dark energy decreases like radiation or matter, for which the small value $\rho_E/M^4 \approx 10^{-120}$ is naturally understood as a consequence of the huge age of the universe in Planck units. The presently observed accelerated expansion requires, however, a recent exit from such a cosmic scaling solution, for example, by growing neutrino quintessence [5, 7, 17, 83, 132] . We will see that a cosmic scaling solution may only be reached very late in the evolution of the universe.

暗能量的详细动力学需要知道 Z 对 φ 的依赖关系。我们将在“穿越宇宙学”一节对此展开讨论。对于常数 $Z < 1/n$ ，我们确实会得到 $\Omega_h = Zn$ 为常数的宇宙标度解 [128]。它们是吸引子: 随着时间增加，邻近解会趋近这些宇宙标度解。宇宙标度解可以自然地解释为何当前值 $\rho_h/M^4 \approx 10^{-120}$ 如此微小。当 Ω_h 为常数时，暗能量和辐射或物质衰减速率相同，因此 $\rho_E/M^4 \approx 10^{-120}$ 的小值可以自然理解为宇宙在普朗克单位下年龄极大的结果。然而，当前观测到的加速膨胀要求最近从这类宇宙标度解退出，例如通过增长中微子精质 [5, 7, 17, 83, 132] 。我们会看到，宇宙标度解可能只有在宇宙演化非常晚期才能达到。

Ultraviolet Fixed Point

紫外不动点

The ultraviolet fixed point corresponds to the limit $\tilde{\rho} \to 0$. For fixed k this is realized for $\chi \to 0$, while for fixed χ it describes $k \to \infty$. For $\tilde{\rho} \to 0$ the effective action according to the scaling solution is approximated by

紫外不动点对应极限 $\tilde{\rho} \to 0$ 。对于固定的 k ，这在 $\chi \to 0$ 下实现，而对于固定的 χ ，它描述 $k \to \infty$ 。对于 $\tilde{\rho} \to 0$ ，标度解对应的有效作用量可近似为

$$\Gamma = \int_x \sqrt{g} \Big\{ -\frac{1}{2}\left(f_0 k^2 + \xi_0 \chi^2\right) R + \frac{1}{2} K \partial^\mu \chi \partial_\mu \chi + u_0 k^4 \Big\} . \tag{41}$$

Due to the leading behavior $F = f_0 k^2, U = u_0 k^4$, the scale k remains present and Γ is not invariant under field scalings (20). In contrast, neglecting the subleading term $\sim \xi_0$ and for

由于主导行为 $F = f_0 k^2, U = u_0 k^4$ ，能标 k 仍然存在，且 Γ 在场标度变换 (20) 下不不变。与之相对，若忽略次领头项 $\sim \xi_0$ 且对于

$$K = \kappa \frac{k^2}{\chi^2}, \tag{42}$$

we observe a different version of quantum scale symmetry where only the scalar field χ is multiplicatively rescaled, $\chi \to \alpha\chi$, while the metric is left invariant. The leading scale symmetry violations close to this fixed point are due to ξ_0 , as well as deviations of K from the form (42) and corrections $\sim m_0^2\chi^2$ for U .

我们观测到量子标度对称性的另一种形式: 其中仅标量场 χ 做乘法重定标 $\chi \to \alpha\chi$ ，而度规保持不变。该不动点附近的主导标度对称性破缺由 ξ_0 ，以及 K 偏离形式 (42)、对应 U 的修正项 $\sim m_0^2\chi^2$ 导致。

One could perform a Weyl scaling with $w^2 = \chi^2/k^2$. This would replace the curvature coefficient by $F' = f_0\chi^2$ and the potential by $U' = u_0\chi^4$, while the factor χ^{-2} in K would no longer be present in K' . In the new metric frame the effective action is invariant under the simultaneous transformations (20) of the metric and the scalar field. The lesson to be learned is that the quantum scale transformations at the IR and UV fixed points need not be the same, or the fields on which they act need not be identical. Quantum scale symmetry at the UV fixed point can actually also be realized if K diverges for $\chi \to 0$ with a power different from χ^{-2} . The renormalized fields with a standard scaling behavior would then be different [138].

我们可以对 $w^2 = \chi^2/k^2$ 做外尔标度变换。这会将曲率系数替换为 $F' = f_0\chi^2$ ，势替换为 $U' = u_0\chi^4$ ，同时 K 中的因子 χ^{-2} 不再出现在 K' 中。在新的度规框架下，有效作用量在度规和标量场的联合变换 (20) 下不变。我们可以得到的结论是: 红外不动点和紫外不动点处的量子标度变换不必相同，或者它们作用的场不必完全相同。若当 $\chi \to 0$ 时 K 以异于 χ^{-2} 的幂次发散，紫外不动点处的量子标度对称性依然可以实现。此时具有标准标度行为的重整化场会有所不同 [138]。

At the UV fixed point the effective action takes a particularly simple form in terms of the scalar field $\tilde{\varphi}$

在紫外不动点处，有效作用量用标量场 $\tilde{\varphi}$ 表示时形式格外简单

$$\tilde{\varphi} = \sqrt{\kappa}k \ln\left(\frac{\chi}{k}\right), \tag{43}$$

namely,

即

$$\Gamma = \int_x \sqrt{g}\left\{-\frac{1}{2}f_0k^2R + u_0k^4 + \frac{1}{2}\partial^\mu\tilde{\varphi}\partial_\mu\tilde{\varphi}\right\}. \tag{44}$$

This describes a massless free scalar field with canonical kinetic term coupled to a form of Einstein gravity with a cosmological constant. The scale transformations act now as shifts in $\tilde{\varphi}$. From Eq. (44) we can obtain the Einstein frame by a constant Weyl scaling with $w^2 = M^2/\left(f_0k^2\right)$, resulting in $U_E = u_0M^4/f_0^2$. We further transform $\tilde{\varphi}$ to $\varphi = 4M\ln(\chi/M)$, resulting in an effective action of type (25) with constant potential and

这描述了一个带正则动能项的无质量自由标量场，耦合到带宇宙学常数的爱因斯坦引力。标度变换现在表现为 $\tilde{\varphi}$ 的平移。我们可以通过对 $w^2 = M^2/\left(f_0k^2\right)$ 做常数外尔标度变换从式 (44) 得到爱因斯坦框架，结果为 $U_E = u_0M^4/f_0^2$ 。我们进一步将 $\tilde{\varphi}$ 变换为 $\varphi = 4M\ln(\chi/M)$ ，得到具有常数势的形式为 (25) 的有效作用量，且

$$Z = \frac{\kappa}{16 f_0} \tag{45}$$

In the Einstein frame the solution of the field equations for $\varphi \to -\infty$ is de Sitter space. (As long as corrections to quantum scale symmetry are not taken into account, we can take an arbitrary constant value for φ as well.) The constant Hubble parameter reads in the Einstein frame

在爱因斯坦框架下，$\varphi \to -\infty$ 场方程的解是德西特空间。(只要不考虑量子标度对称性的修正，我们也可以取 φ 为任意常数。) 爱因斯坦框架下的常数哈勃参数为

$$H_E^2 = \frac{u_0 M^4}{3 f_0^2}. \tag{46}$$

De Sitter space is a good approximation for the inflationary epoch of the universe.

德西特空间是宇宙暴胀时期的良好近似。

The primordial fluctuations of the scalar and graviton (traceless transverse tensor of the metric fluctuations) are given by the propagators of the respective fields. In turn, these propagators are determined as the inverse of the second functional derivative of the quantum effective action [141]. For de Sitter space one finds an exactly scale-invariant primordial fluctuation spectrum (spectral index $n_s = 0$). This scale invariance is directly rooted in the quantum scale symmetry of the effective action. No intrinsic parameter with dimension of mass or length appears in the effective action once it is expressed in terms of appropriate renormalized fields. This explains why no scale appears in the fluctuation spectrum. The spectrum of the primordial cosmic fluctuations does not depend on the metric frame [139].

标量和引力子 (度规涨落的无迹横向张量) 的原初涨落由对应场的传播子给出，而这些传播子由量子有效作用量二阶泛函导数的逆确定 [141]。对于德西特空间可以得到精确标度不变的原初涨落谱 (谱指数 $n_s = 0$)。这种标度不变性直接源于有效作用量的量子标度对称性。当有效作用量用合适的重整化场表示后，不会出现任何质量或长度量纲的内禀参数，这解释了涨落谱中为何不出现尺度。原初宇宙涨落的谱不依赖于度规框架 [139]。

The amplitude of the graviton fluctuations obeys the frame-invariant expression [139, 141]

引力子涨落的振幅满足洛伦兹不变式 [139, 141]

$$\Delta_T^2 = \frac{2\hat{\mathcal{H}}^2}{\pi^2 A^2} = \frac{2H_E^2}{\pi^2 M^2} = \frac{2U_E}{3\pi^2 M^4}, \tag{47}$$

where the last two equations insert the values for the Einstein frame. We may compare with the observed amplitude of the cosmic fluctuation spectrum for scalar fluctuations [2]

最后两式代入了爱因斯坦框架的取值。我们可以和观测到的标量涨落宇宙涨落谱振幅进行对比 [2]

$$\mathcal{A} = \frac{3\pi^2}{2r}\Delta_T^2 = 3.56 \cdot 10^{-8}, \tag{48}$$

with tensor to scalar ratio $r < 0.05$ [1]. This limits the value of the potential at the time when the primordial fluctuations are frozen,

张标比为 $r < 0.05$ [1]。这限制了原初涨落冻结时势能的取值,

$$r\mathcal{A} = \frac{U_E}{M^4} = \hat{V} = \frac{u}{f^2}. \tag{49}$$

Very close to the UV fixed point, this would entail the constraint $u_0/f_0^2 = r\mathcal{A} \lesssim 2 \cdot 10^{-9}$. We will see in the next section that such a small value seems rather unlikely to result from a quantum gravity computation of u_0 and f_0 . This is an example of typical restrictions following from quantum gravity. One concludes that the decoupling of the observed fluctuations should occur at a later time when H_E is already substantially smaller than the value very close to the fixed point. This will be discussed in section ”Crossover Cosmology.”

非常接近紫外固定点时，这会给出约束 $u_0/f_0^2 = r\mathcal{A} \lesssim 2 \cdot 10^{-9}$ 。我们将在下一节看到，u_0 和 f_0 的量子引力计算不太可能得到这么小的值。这是量子引力带来典型限制的一个例子。由此可得结论，观测到的涨退退耦应当发生在更晚的时间，此时 H_E 已经远小于非常接近固定点处的取值。我们将在“交越宇宙学”一节讨论这一内容。

Flow Equations for Quantum Gravity

量子引力的流方程

This section presents the functional flow equation on which our estimates of the properties of the scaling solution and its limiting behavior for $\tilde{\rho} \to 0$ and $\tilde{\rho} \to \infty$ are based. We work in second order in a derivative expansion. The scale-dependent effective action Γ_k is therefore truncated to the form (3). The flow equations are evaluated for the Euclidean signature of the metric. Analytic continuation to Minkowski signature does not seem to pose any major problem at this level since all inverse propagators have the form $Zq^2 + m^2$. We report on the flow equations for u and f and discuss properties of the flow equation for K .

本节介绍我们对标度解性质及其在 $\tilde{\rho} \to 0$ 和 $\tilde{\rho} \to \infty$ 下极限行为进行估计所依据的泛函流方程。我们在导数展开的二阶框架下开展工作，因此依赖标度的有效作用量 Γ_k 被截断为形式 (3)。我们在欧几里得度规符号下计算流方程。在此层面，解析延拓到闵氏度规符号似乎不存在重大问题，因为所有逆传播子都具有形式 $Zq^2 + m^2$ 。本文给出 u 和 f 的流方程，并讨论 K 流方程的性质。

Diffeomorphism Invariant Flow Equation for Quantum Gravity

量子引力的微分同胚不变流方程

The functional flow equation for the effective average action Γ_k and its adaption to gauge theories and gravity has been developed in Ref. [103, 105, 149]. We report here on the gauge-invariant formulation [142] which offers both technical simplifications and a direct connection to observable quantities. In this formulation the first functional derivative of $\Gamma_{k\to 0}$ yields the field equations for cosmology, while the second functional derivative determines the inverse propagator. One obtains the propagator and thereby the fluctuation spectrum by inversion. In case of fundamental scale invariance, the field equations and propagators can be computed for an arbitrary choice of k .

有效平均作用量 Γ_k 的泛函流方程及其对规范理论和引力的适配已在文献 [103, 105, 149] 中完成推导。本文在此介绍规范不变表述 [142]，该表述兼具技术简化性与可观测物理量的直接关联。在此表述中，$\Gamma_{k\to 0}$ 的一阶泛函导数给出宇宙学场方程，二阶泛函导数则确定逆传播子，通过求逆即可得到传播子，进而得到涨落谱。在基本标度不变的情况下，对 k 的任意选取都可计算场方程与传播子。

The exact flow equation takes the simple one-loop form,

精确流方程可写为简洁的单圈形式:

$$k\partial_k\Gamma_k = \frac{1}{2}\,\mathrm{Str}\left\{\left(\Gamma_k^{(2)} + R_k\right)^{-1} k\partial_k R_k\right\} - \delta_k. \tag{50}$$

Here $\left(\Gamma_k^{(2)} + R_k\right)^{-1}$ is the full propagator in the presence of arbitrary ”macroscopic fields” and the infrared regulator R_k . Thus, both Γ_k and $\Gamma_k^{(2)}$ are functionals of these fields and Eq. (50) is a functional differential equation. In momentum space the supertrace Str contains a momentum integral $\int_q = \int \mathrm{d}^4q/(2\pi)^4$, a sum over different species of particles with a minus sign for fermions, as well as a trace over internal indices, including Lorentz indices μ, ν or spinor indices if appropriate. The cutoff vanishes rapidly for squared momenta $q^2 \gg k^2$ such that the momentum integral is ultraviolet finite due to the decay of $k\partial_k R_k$. Infrared finiteness is assured by the presence of the regulator term in the inverse propagator $\Gamma_k^{(2)} + R_k$. With a UV-and IR-finite right-hand side, there is no need for an additional UV regularization. Once the flow equation is established, one needs no more an explicit regularized functional integral respecting the symmetries. The microphysics is encoded in the ”initial conditions” of Γ_k for $k \to \infty$. This implicit ”ERGE regularization” constitutes an important advantage for theories for which no explicit gauge-invariant regularization is known, as in the case of quantum gravity. Finally, δ_k is a ”measure factor” which accounts for the redundant formulation in case of local gauge theories.

此处 $\left(\Gamma_k^{(2)} + R_k\right)^{-1}$ 是存在任意” 宏观场” 和红外调节器 R_k 时的完整传播子。因此, Γ_k 和 $\Gamma_k^{(2)}$ 均为这些场的泛函,式 (50) 是一个泛函微分方程。在动量空间中,supertrace Str 包含动量积分 $\int_q = \int \mathrm{d}^4q/(2\pi)^4$ 、对不同粒子种类的求和 (费米子带负号)，以及对内部指标的迹，包括洛伦兹指标 μ, ν (若适用还包括旋量指标)。对平方动量 $q^2 \gg k^2$ ，截断快速衰减，因此由于 $k\partial_k R_k$ 的衰减特性，动量积分是紫外有限的。逆传播子 $\Gamma_k^{(2)} + R_k$ 中的调节器项保证了红外有限性。由于方程右侧同时满足紫外有限与红外有限，无需额外引入紫外正则化。一旦确立流方程，便不再需要满足对称性的显式正则化泛函积分。微观物理编码在 Γ_k 对应 $k \to \infty$ 的” 初值条件” 中。这种隐式的”ERGE 正则化” 对不存在显式规范不变正则化的理论 (比如量子引力) 构成了重要优势。最后，δ_k 是” 测度因子”，用于处理局部规范理论中的冗余表述。

For the gauge-invariant formulation of the flow equation, the first term on the r.h.s. involves a projection on the physical fluctuations. This is effectively achieved by a suitable ”physical gauge fixing.” In this formulation the conceptual structure of the first term is completely analogous to simpler theories for scalars and fermions without local gauge invariance. The ”measure factor” is given by a simple functional (typically a derivative of a regularized determinant) that does not depend on Γ_k . Ghosts need not to be introduced for this purpose since the Faddeev-Popov determinant can be regularized directly. We will not describe here all computational steps leading to the flow equations for u, f , and K . We only present the main lines and the results which have a simple intuitive form.

对于流方程的规范不变表述，方程右侧第一项包含对物理涨落的投影，这可通过合适的” 物理规范固定” 有效实现。该表述下，第一项的概念结构与不存在局部规范不变性的标量、费米子这类更简单理论完全类似。测度因子由一个不依赖 Γ_k 的简单泛函给出 (通常是正则化行列式的导数)。因此无需引入鬼场，因为法捷耶夫-波波夫行列式可以直接正则化。本文不赘述推导出 u, f 和 K 流方程的全部计算步骤，仅介绍核心思路与具有简洁直观形式的结果。

Flow Equation for Effective Potential

有效势的流方程

We first evaluate the flow equation for constant scalar fields and a constant metric. Since all derivatives vanish, this projects Γ_k on the effective potential for the scalar fields, multiplied by $\sqrt{g}$, i.e., $\Gamma_k = \int_x \sqrt{g} U_k$. Correspondingly, $\Gamma_k^{(2)}$ has to be evaluated for constant macroscopic fields. In the presence of these fields, one typically finds momentum-independent contributions to $\Gamma_k^{(2)}$. These field-dependent ”mass terms” are functions of the constant scalar fields.

我们首先对标量场常数、度规常数的情况计算流方程。由于所有导数都为零，这会将 Γ_k 投影到标量场的有效势上，再乘以 $\sqrt{g}$，即得到 $\Gamma_k = \int_x \sqrt{g} U_k$。相应地，$\Gamma_k^{(2)}$ 必须在宏观场常数的条件下计算。存在这类场时，我们通常会得到 $\Gamma_k^{(2)}$ 的动量无关贡献。这些依赖场的“质量项”都是常数标量场的函数。

The flow equation for the effective scalar potential U_k can be written in an intuitive form [91,144]

有效标量势 U_k 的流方程可以写成直观形式 [91,144]

$$k\partial_k U_k = \widetilde{\pi}_U = \widetilde{\pi}_{\text{grav}} + \widetilde{\pi}_s + \widetilde{\pi}_{\text{gauge}} + \widetilde{\pi}_f$$

$$= \frac{k^4}{32\pi^2} \left(2\bar{N}_g + \bar{N}_S + 2\bar{N}_V - 2\bar{N}_F \right) = 4k^4 c_U. \tag{51}$$

Different parts arise from fluctuations of different fields, with $\bar{N}_j$ the effective numbers of particle species as described below.

不同部分来自不同场的涨落，其中 $\bar{N}_j$ 是下文所述粒子种类的有效数目。

Metric Fluctuations

度规涨落

The first contribution arises from the metric fluctuations

第一份贡献来自度规涨落

$$\widetilde{\pi}_{\text{grav}} = \frac{k^4}{24\pi^2}\left(1-\frac{\eta_g}{8}\right)\left(\frac{5}{1-v}+\frac{1}{1-v/4}\right)-\frac{k^4}{8\pi^2}. \tag{52}$$

It depends on the dimensionless ratio

它取决于无量纲比值

$$v = \frac{2U}{Fk^2} = \frac{2u}{f}. \tag{53}$$

Here the first term in Eq. (52) reflects the five degrees of freedom of the traceless tensor fluctuations (graviton fluctuations) whose propagator involves an effective mass term $-2U/F$. The second term is due to the physical scalar degree in the metric fluctuation with effective mass term $-U/(2F)$. (Here physical fluctuations are defined in contrast to the pure gauge fluctuations. This does not mean that the physical scalar metric fluctuation, which accounts for Newton's potential, is propagating as a particle. The particle degrees of freedom are only two polarizations of the graviton.) Finally, the constant last term reflects the metric contribution to the measure factor δ_k .

式 (52) 中第一项反映无迹张量涨落 (引力子涨落) 的五个自由度，其传播子包含有效质量项 $-2U/F$ 。第二项来自度规涨落中带有有效质量项 $-U/(2F)$ 的物理标量自由度。(此处物理涨落是相对于纯规范涨落定义的，这并不意味着描述牛顿势的物理标量度规涨落会作为粒子传播，粒子自由度只有引力子的两个偏振态。) 最后，恒定的末项反映度规对测度因子 δ_k 的贡献。

For the precise form of the flow equations, we follow Ref. [91]; for early investigations, see Ref. [34, 42, 74, 86]. We have taken a particular form of the infrared cutoff function, namely, a Litim-type regulator [79] $R_k \sim (k^2-q^2)\,\theta(k^2-q^2)$. This replaces an inverse propagator q^2 by k^2 if $q^2 < k^2$ and does not change the propagator for $q^2 > k^2$, leading to $k\partial_k R_k = 0$ for $q^2 > k^2$. For this regulator a mass term m^2 in the inverse propagator $\sim (q^2+m^2)$ generates a "threshold function"

关于流方程的精确形式，我们遵循文献 [91]；早期研究参见文献 [34, 42, 74, 86]。我们采用了一种特殊形式的红外截断函数，即 Litim 型调节项 [79] $R_k \sim (k^2-q^2)\,\theta(k^2-q^2)$。该调节项将逆传播子 q^2 替换为 k^2 (当 $q^2 < k^2$ 时)，且不改变 $q^2 > k^2$ 的传播子，由此得到对应 $q^2 > k^2$ 的 $k\partial_k R_k = 0$ 。对于该调节项，逆传播子 $\sim (q^2+m^2)$ 中的质量项 m^2 会生成一个“阈函数”

$$s(\widetilde{m}^2) = (1+\widetilde{m}^2)^{-1},\ \widetilde{m}^2 = \frac{m^2}{k^2}, \tag{54}$$

which multiplies the contribution of the massive particle. This threshold function leads to an automatic suppression of the contribution from particles with $m^2 > k^2$, such that the flow equation incorporates naturally the decoupling of heavy particles. This contrasts to many other regularization schemes as dimensional regularization. For other choices of the regulator R_k , the precise form of the threshold function will differ, but the qualitative decoupling behavior remains the same.

该阈函数乘以大质量粒子的贡献。它会自动抑制满足 $m^2 > k^2$ 的粒子的贡献，因此流方程自然包含了重粒子的退耦效应，这和维度正则化等许多其他正则化方案不同。若选择其他调节项 R_k ，阈函数的精确形式会发生变化，但定性的退耦行为保持不变。

We also observe a pole in the threshold function for $\tilde{m}^2 \to -1$. This can be related to convexity properties of the effective potential [120]. For the graviton contribution the factor $(1-v)^{-1}$ reflects this threshold function, with $\tilde{m}^2 = -v$, and similar for the scalar metric fluctuation with $\tilde{m}^2 = -v/4$. We note that the effective mass term $\tilde{m}^2$ for the metric fluctuations is negative for positive u and f . Values of v close to one can substantially enhance the impact of the graviton fluctuations. They dominate for the range of positive v .

我们还观测到当 $\tilde{m}^2 \to -1$ 时阈函数中存在一个极点，这和有效势的凸性性质相关 [120]。对于引力子贡献，因子 $(1-v)^{-1}$ (满足 $\tilde{m}^2 = -v$) 反映了该阈函数，带有 $\tilde{m}^2 = -v/4$ 的标量度规涨落也类似。我们注意到，对于度规涨落，当 u 和 f 为正时，有效质量项 $\tilde{m}^2$ 为负。接近 1 的 v 会大幅增强引力子涨落的影响，在 v 为正的区间，引力子涨落占主导。

Finally, the quantity η_g ,

最后，量 η_g

$$\eta_g = -k\partial_k \ln f = 2 - \frac{k\partial_k F}{F}, \tag{55}$$

reflects that the regulator for the metric fluctuations is taken proportional to F . Thus, η_g vanishes for constant f and equals two if F is independent of k . In the limit $|v| \ll 1$ one finds for constant F

反映出度规涨落的调节项与 F 成正比。因此，当 f 为常数时，η_g 等于零；若 F 独立于 k ，则 η_g 等于 2。在极限 $|v| \ll 1$ 下，对于常数 F 可得

$$\tilde{\pi}_{\text{grav}} = \frac{k^4}{16\pi^2}, \; \bar{N}_g = 1. \tag{56}$$

This corresponds to the contribution of the two propagating degrees of freedom of the graviton. In our setting for cosmology this limit applies for $k^2 \ll \chi^2$.

这对应引力子两个传播自由度的贡献。在我们的宇宙学设定中，当 $k^2 \ll \chi^2$ 时该极限成立。

Scalar Fluctuations

标量涨落

The contribution from scalar fluctuations is the same as for models without gravity

标量涨落的贡献与无引力模型中的贡献相同

$$\tilde{\pi}_s = \frac{k^4}{32\pi^2} \sum_A \left(1 - \frac{\eta_A}{6}\right)\left(1 + \tilde{m}_A^2\right)^{-1} = \frac{\bar{N}_S k^4}{32\pi^2}, \tag{57}$$

where the sum runs over N_S scalar fields. The index A labels the eigenvalues m_A^2 of the renormalized scalar mass matrix M^2 ,

其中求和对 N_S 个标量场进行。指标 A 标记重整化标量质量矩阵 M^2 的本征值 m_A^2 ,

$$M_{ab}^2 = (Z_a Z_b)^{-1/2} \frac{\partial^2 U}{\partial\phi_a \partial\phi_b},\ \widetilde{m}_A^2 = \frac{m_A^2}{k^2}. \tag{58}$$

Here Z_a is the kinetial of the scalar field $\phi_a, a = 1 \ldots N_S, \eta_a = -k\partial_k \ln Z_a$. The anomalous dimension η_A arises from Z_A multiplying R_k and is identified with some suitable η_a . It is typically a small quantity and can be neglected. We identify $\bar{N}_S$ in Eq. (51),(57) with the effective number of real scalar fields. For $\eta_A = 0$ it coincides with the number of effectively massless scalars for which $\widetilde{m}_A^2 \ll 1$. Since $\widetilde{m}_A^2$ depends on the values of the constant macroscopic scalar fields, the effective number $\bar{N}_S$ varies in different regions of field space and for different k . In between mass thresholds one finds, however, an (almost) constant value of $\bar{N}_S$. The overall picture is simple: every effectively massless scalar contributes to $\widetilde{\pi}_s$ a term $k^4/\left(32\pi^2\right)$. For massless scalars the only mass scale is given by k , such that the factor k^4 is dictated by the dimension of the scalar potential U_k .

此处 Z_a 是标量场 $\phi_a, a = 1 \ldots N_S, \eta_a = -k\partial_k \ln Z_a$ 的动能项。反常维度 η_A 由 Z_A 乘 R_k 产生，对应某个合适的 η_a 。它通常是一个小量，可以忽略。我们将式 (51)、(57) 中的 $\bar{N}_S$ 认定为实标量场的有效数目。对于 $\eta_A = 0$ ，它与满足 $\widetilde{m}_A^2 \ll 1$ 的有效无质量标量的数目一致。由于 $\widetilde{m}_A^2$ 依赖于宏观常数标量场的取值，有效数目 $\bar{N}_S$ 在场空间的不同区域以及不同 k 下会发生变化。但在质量阈值之间，$\bar{N}_S$ 会取到一个 (几乎) 恒定的值。整体图像很简单: 每个有效无质量标量都会对 $\widetilde{\pi}_s$ 贡献一项 $k^4/\left(32\pi^2\right)$ 。对于无质量标量，唯一的质量尺度由 k 给出，因此因子 k^4 由标量势 U_k 的维度决定。

The flow equation (51), (52), (57) is derived in the truncation of variable gravity (3) which includes terms with up to two derivatives. Within this truncation we have omitted a subleading term. For $\partial U/\partial\phi = 0$ the scalar degree of freedom in the metric fluctuations mixes with the other scalars ϕ_a . The resulting correction term [91] $\sim (\partial U/\partial\varphi)^2$ vanishes at the minimum of U and can be neglected for sufficiently flat U .

流方程 (51)、(52)、(57) 是在可变引力 (3) 的截断下推导得到的，该截断包含最高到二阶导数的项。在此截断内我们省略了一个次领头项。对于 $\partial U/\partial\phi = 0$ ，度规涨落中的标量自由度会与其他标量 ϕ_a 混合。由此产生的修正项 [91] $\sim (\partial U/\partial\varphi)^2$ 在 U 的极小值处为零，对于足够平缓的 U 可以忽略。

We evaluate $U(\chi)$ at the partial minimum with respect to other additional scalar fields as the Higgs doublet. At the partial minimum these additional scalar fields do not mix with χ through the mass matrix. They also do not mix with the metric fluctuations. Then the additional scalars decouple in a range of k smaller than their masses. These properties single out the definition of χ at the partial minimum with respect to the other scalars. For a different choice the flow equations would be more complicated and do not feature the effective decoupling. (Our flow equations concern the effective potential at zero temperature. If fields are displaced from their minimum in vacuum due to temperature effects, the situation gets more complex.) For k smaller than the mass of the lightest additional scalar, only the fluctuations of χ contribute to $\widetilde{\pi}_s$, with

我们就像希格斯二重态那样，对额外标量场在偏极小值处计算 $U(\chi)$ 。在偏极小值处，这些额外标量场不会通过质量矩阵与 χ 混合，也不会与度规涨落混合。在 k 小于它们质量的范围内，额外标量会退耦。这些性质明确了在对其他标量取偏极小值时 χ 的定义。如果选择不同的偏极小值，流方程会变得更复杂，也不存在有效退耦。(我们的流方程针对零温下的有效势。如果由于温度效应，场偏离了真空极小值，情况会更复杂。) 当 k 小于最轻额外标量的质量时，只有 χ 的涨落对 $\tilde{\pi}_s$ 有贡献，满足

$$\tilde{m}^2 = \frac{\partial u}{\partial \tilde{\rho}} + 2\frac{\partial^2 u}{\partial \tilde{\rho}^2}. \tag{59}$$

In the region where $u(\tilde{\rho})$ is flat, one has approximately $\tilde{m}^2 = 0$, and therefore, $\bar{N}_S = 1 - \eta_s/6$. The mixing with the metric fluctuations due to $(\partial U/\partial\chi)^2$ can be neglected in these regions.

在 $u(\tilde{\rho})$ 平缓的区域，近似有 $\tilde{m}^2 = 0$ ，因此得到 $\bar{N}_S = 1 - \eta_s/6$ 。在这些区域，由 $(\partial U/\partial\chi)^2$ 引起的与度规涨落的混合可以忽略。

Fermion and Gauge Boson Fluctuations

费米子与规范玻色子涨落

The contributions from fermion fluctuations is even simpler. For effectively massless fermions $\bar{N}_F$ counts the number of Weyl fermions or equivalently Majorana fermions. For example, in the region of $k \gg m_e$, the electron fluctuations contribute $\bar{N}_{F,e} = 2$ as appropriate for a Dirac fermion which is constituted of two Majorana fermions or Weyl fermions. In general, we consider N_F Majorana fermions with masses m_f^2 and $\tilde{m}_f^2 = m_f^2/k^2$. Neglecting possible small anomalous dimensions for the fermionic kinetic terms, they contribute

费米子涨落的贡献更为简单。对于有效无质量费米子，$\bar{N}_F$ 对应外尔费米子的数量，等价于马约拉纳费米子的数量。例如，在 $k \gg m_e$ 区域，电子涨落的贡献为 $\bar{N}_{F,e} = 2$ ，符合狄拉克费米子的情况——狄拉克费米子由两个马约拉纳费米子或两个外尔费米子构成。一般情况下，我们考虑质量为 m_f^2 和 $\tilde{m}_f^2 = m_f^2/k^2$ 的 N_F 个马约拉纳费米子。忽略费米子动能项可能存在的小反常维数，它们的贡献为

$$\tilde{\pi}_f = -\frac{\bar{N}_F k^4}{16\pi^2},\ \bar{N}_F = \sum_{f=1}^{N_f} \left(1 + \tilde{m}_f^2\right)^{-1}. \tag{60}$$

We observe again the decoupling of fermions once $m_f^2 \ll k^2$. For the scaling solution the fermion masses are given by effective dimensionless Yukawa couplings h_f ,

我们再次观测到当 $m_f^2 \ll k^2$ 时费米子退耦。对于标度解，费米子质量由有效无量纲汤川耦合 h_f 给出，

$$m_f = h_f\chi,\ \tilde{m}_f^2 = 2h_f^2\tilde{\rho}. \tag{61}$$

For the contribution $\tilde{\pi}_{\text{gauge}}$ from gauge bosons, $\bar{N}_V$ counts the number of effectively massless gauge bosons. For each massless gauge boson the physical fluctuations are the transversal fluctuations and contribute a factor three. The measured term subtracts one, resulting in the expression $2\bar{N}_V$ in Eq. (51), which reflect the two polarizations of a propagating massless vector field. Gauge bosons can acquire masses m_v through the Higgs mechanism, $\tilde{m}_v^2 = m_v^2/k^2$, such that $\bar{N}_V$ is approximated by

对于规范玻色子的贡献 $\tilde{\pi}_{\text{gauge}}$ ，$\bar{N}_V$ 对应有效无质量规范玻色子的数量。每个无质量规范玻色子的物理涨落为横涨落，贡献因子 3。测量项减去 1 后，得到式 (51) 中的表达式 $2\bar{N}_V$ ，该表达式反映了传播中的无质量矢量场的两种偏振。规范玻色子可以通过希格斯机制 $\tilde{m}_v^2 = m_v^2/k^2$ 获得质量 m_v ，因此 $\bar{N}_V$ 可近似为

$$2\bar{N}_V = \sum_{v=1}^{N_V} \left[3\left(1 + \tilde{m}_v^2\right)^{-1} - 1 \right] \tag{62}$$

The three massive gauge boson degrees of freedom decouple for $\tilde{m}_v^2 \gg 1$. What remains is the measure factor which does not involve $\tilde{m}_v^2$. This measure factor cancels precisely the contribution of the massless Goldstone boson in $\bar{N}_S$. For each massive gauge boson there is one massless Goldstone boson that transmutes to the longitudinal massive gauge boson. As a result, only the three degrees of freedom of the massive gauge boson and the massive non-Goldstone scalar modes (”radial modes”) contribute to the flow. They decouple once all mass terms exceed k^2 .

当 $\tilde{m}_v^2 \gg 1$ 时，有质量规范玻色子的三个自由度全部退耦，仅剩不包含 $\tilde{m}_v^2$ 的测度因子。该测度因子恰好抵消了 $\bar{N}_S$ 中无质量戈德斯通玻色子的贡献。每个有质量规范玻色子对应一个无质量戈德斯通玻色子，后者转化为纵向有质量规范玻色子。因此，只有有质量规范玻色子的三个自由度和有质量非戈德斯通标量模式 (“径向模式”) 对流有贡献。当所有质量项都超过 k^2 时，这些自由度就会退耦。

For the quantum scale-invariant standard model, the masses of the W- and Z-bosons are proportional to the Fermi scale φ_0 , which in turn is proportional to χ . This results in $\tilde{m}_v^2 = c_v\tilde{\rho}$, with very small $c_v \sim g^2\varphi_0^2/\chi^2$ involving the gauge coupling g and the tiny ratio φ_0^2/χ^2 . This ensures that the W- and Z-bosons decouple only once k gets smaller than their mass.

对于量子标度不变的标准模型，W 玻色子和 Z 玻色子的质量与费米能标 φ_0 成正比，而费米能标又与 χ 成正比。由此得到 $\tilde{m}_v^2 = c_v\tilde{\rho}$ ，其中包含规范耦合常数 g 和极小比值 φ_0^2/χ^2 的 $c_v \sim g^2\varphi_0^2/\chi^2$ 非常小。这保证了只有当 k 小于 W 玻色子和 Z 玻色子的质量时，二者才会退耦。

Robustness of Flow Equation

流方程的鲁棒性

In summary, a rough approximation to the flow of U_k simply counts the degrees of freedom for massless particles, consisting of $\bar{N}_g$ gravitons, $\bar{N}_S$ scalars, $\bar{N}_F$ fermions, and $\bar{N}_V$ gauge bosons, as relevant for a given range of k or $\tilde{\rho}$. In view of this very simple structure, where only the physical propagating modes contribute in the range where their masses are smaller than k , the flow equation for U seems to be rather robust.

总而言之，U_k 流的粗略近似仅统计无质量粒子的自由度，在 k 或 $\tilde{\rho}$ 的给定相关范围内，包含 $\bar{N}_g$ 个引力子、$\bar{N}_S$ 个标量场、$\bar{N}_F$ 个费米子和 $\bar{N}_V$ 个规范玻色子。鉴于这种极为简单的结构——只有质量小于 k 的物理传播模式才会对该范围产生贡献，U 的流方程似乎具有相当强的鲁棒性。

In the gauge-invariant formulation of the flow equation, propagators and vertices are obtained by taking appropriate derivatives of Γ_k . From the potential U_k we obtain the mass matrix by taking two derivatives:

在流方程的规范不变表述中，传播子和顶点可通过对 Γ_k 求适当导数得到。我们可以对势场 U_k 求二阶导得到质量矩阵:

$$\bar{M}_{ab}^2 = \frac{\partial^2 U}{\partial\varphi_a \partial\varphi_b}. \tag{63}$$

Correspondingly, the flow equation for $\bar{M}_{ab}^2$ is found by taking two derivatives of the flow generator

相应地，通过对流生成元求二阶导，即可得到 $\bar{M}_{ab}^2$ 的流方程

$$k\partial_k \bar{M}_{ab}^2 = \frac{\partial^2 \widetilde{\pi}_U}{\partial\varphi_a \partial\varphi_b}. \tag{64}$$

This generalizes to the flow of vertices, as quartic scalar couplings which involve four derivatives of U . Omitting the contributions of metric fluctuations, which is suppressed for $k^2 \ll F$, this procedure reproduces the perturbative β -functions for the running quartic couplings in one-loop order, plus part of the higher-loop contributions. Exact two-loop β -functions require an extended truncation [88]. It is interesting to note that it is precisely the threshold functions for the decoupling of massive particles which are responsible for the running quartic couplings. The field dependence of $k\partial_k U_k$ arises only from these threshold functions which induce a field dependence of $\bar{N}_S, \bar{N}_V, \bar{N}_F$. The one-loop β -functions are universal in the sense that they do not depend on the choice of the IR cutoff R_k . The fact that perturbative β -functions were obtained in a straightforward way from the flow equation (51) enhances our confidence in the validity of this approach.

这可以推广到顶点的流，比如涉及 U 四阶导数的四次标量耦合。忽略度量涨落的贡献 (该贡献在 $k^2 \ll F$ 下被抑制)，该过程可以重现单圈阶跑动四次耦合的微扰 β 函数，以及部分多圈贡献。精确的两圈 β 函数需要扩展截断 [88]。值得注意的是，正是大质量粒子退耦的阈值函数决定了跑动四次耦合。$k\partial_k U_k$ 的场依赖性完全来自这些阈值函数，这些阈值函数也诱导了 $\bar{N}_S, \bar{N}_V, \bar{N}_F$ 的场依赖性。单圈 β 函数具有普适性，它们不依赖于红外截断 R_k 的选择。我们可以从流方程 (51) 直接得到微扰 β 函数，这一事实增强了我们对该方法有效性的信心。

Flow Equation for Curvature Coefficient

曲率系数流方程

For extracting the flow equation for F , we continue to take constant scalar fields. In contrast, we consider a metric $g_{\mu\nu}(x)$ different from the constant metric for flat space. We choose this metric such that the associated curvature scalar R is small. By evaluating the difference of the flow of Γ_k as compared to the flow in flat space, one extracts the flow of $-\frac{1}{2}\int_x \sqrt{g} FR$ and therefore F or f . For the evaluation of the flow generator, one may use heat kernel methods for general metrics or specialize to particular metric configurations as the ones for spheres.

为了推导出 F 的流方程，我们继续假设标量场为常数。与此不同，我们考虑的度规 $g_{\mu\nu}(x)$ 不同于平坦空间的常数度规。我们选取该度规使得关联曲率标量 R 为小量。通过计算 Γ_k 的流与平坦空间中流的差值，我们可以提取出 $-\frac{1}{2}\int_x \sqrt{g} F R$ 的流，进而得到 F 或 f 的流。计算流生成元时，可以对一般度规使用热核方法，也可以对球面这类特殊度规构型做简化处理。

Following Ref. [154] the gauge-invariant flow equation leads to

根据文献 [154]，规范不变流方程给出

$$
\begin{aligned}
k\partial_k F = 2k^2 c_F &= 2k^2\left(c_F^{(\mathrm{grav})} + c_F^{(S)} + c_F^{(F)} + c_F^{(V)}\right) \\
&= 2k^2 c_F^{(\mathrm{grav})} + \frac{k^2}{48\pi^2}\left(-\bar{N}_S - \bar{N}_F + 4\bar{N}_V'\right).
\end{aligned} \tag{65}
$$

The contribution from metric fluctuations is given by

度规涨落的贡献为

$$
c_F^{(\mathrm{grav})} = \frac{25\left(1-\eta_g/6\right)}{64\pi^2(1-v)} - \frac{\left(1-11\eta_g/64\right)}{72\pi^2(1-v/4)} + \frac{17}{192\pi^2}. \tag{66}
$$

The first term arises from the traceless tensor or graviton fluctuations, the second approximates the contribution of the scalar metric fluctuations, and the last term reflects the measure contribution for the metric sector. We observe that the graviton fluctuations dominate over the scalar metric fluctuations by a large factor (unless v takes very large negative values). We have again simplified the scalar sector by omitting the mixing of the scalar metric fluctuations with additional scalar fields. (For an explicit expression for this small correction, see Ref. [154].)

第一项来自无迹张量即引力子涨落，第二项近似给出标量度规涨落的贡献，最后一项反映度规部分的测度贡献。我们可以看到，引力子涨落远大于标量度规涨落 (除非 v 取非常大的负值)。我们再次简化了标量 sector，省略了标量度规涨落与额外标量场的混合。(关于这一小修正的显式表达式参见文献 [154]。)

The contribution from scalar fluctuations is given by

标量涨落的贡献为

$$
c_F^{(S)} = -\frac{\bar{N}_S}{96\pi^2} + c_F^{(\xi)}, \tag{67}
$$

with an effective number of massless scalars $\bar{N}_S$ given by Eq. (57). The second term arises from the field dependence of the curvature coefficient. For a single scalar field with inverse propagator (neglecting mixing with the scalar metric fluctuation),

其中无质量标量的有效数目 $\bar{N}_S$ 由式 (57) 给出。第二项来自曲率系数的场依赖关系。对于单个标量场，其逆传播子 (忽略与标量度规涨落的混合) 为

$$
\begin{aligned}
G^{-1} &= Zq^2 + \frac{\partial^2 U}{\partial \chi^2} - \frac{1}{2}\frac{\partial^2 F}{\partial \chi^2} R \\
&= Z\left(q^2 + m^2 - \tilde{\xi} R\right),
\end{aligned} \tag{68}
$$

where $\tilde{\xi} = \left(\partial^2 F / \partial \chi^2\right) / (2Z)$ one finds [154]

其中 $\tilde{\xi} = \left(\partial^2 F / \partial \chi^2\right) / (2Z)$ ，文献 [154] 给出

$$
c_F^{(\xi)} = -\frac{\tilde{\xi}}{32\pi^2 (1 + \tilde{m}^2)^2}. \tag{69}
$$

In turn, the flow equation for $\tilde{\xi}$ is obtained by taking the second χ -derivative of Eq. (65); see Ref. [86] for an early computation. Off-diagonal kinetic terms mixing the scalar metric fluctuations with the fluctuations of the additional scalar render the situation more complex. We will in the following omit this contribution, keeping in mind that a better understanding is needed.

对式 (65) 取二阶 χ 导数，即可得到 $\tilde{\xi}$ 的流方程；早期计算可参见文献 [86]。非对角动能项混合了标量度规涨落与额外标量的涨落，使得情况更为复杂。在下文中我们将省略这一贡献，但需注意该问题仍有待更好的理解。

The fermion contribution obtains as

费米子贡献为

$$
c_F^{(F)} = -\frac{\bar{N}_F}{96\pi^2}, \tag{70}
$$

with $\bar{N}_F$ given by Eq. (60). For the contribution of gauge boson fluctuations, one finds

其中 $\bar{N}_F$ 由式 (60) 给出。关于规范玻色子涨落的贡献，可得

$$
c_F^{(V)} = \frac{4\bar{N}_V'}{96\pi^2}, \; 4\bar{N}_V' = \sum_v \left[3\left(1 + \tilde{m}_v^2\right)^{-1} + 1\right]. \tag{71}
$$

The last constant term is the measure contribution from the gauge sector for which we note the opposite sign as for the contribution to the flow of the effective potential. This contribution cancels the contribution of the Goldstone boson from $c_F^{(S)} \sim -\bar{N}_S$. Again, massive gauge bosons decouple in the limit $\tilde{m}_v^2 \gg 1$, with only the three massive degrees of freedom contributing.

最后那个常数项是规范 sector 的测度贡献，我们注意到它的符号与有效势流的贡献符号相反。该贡献抵消了来自 $c_F^{(S)} \sim -\bar{N}_S$ 的戈德斯通玻色子贡献。同样，在 $\tilde{m}_v^2 \gg 1$ 极限下，大质量规范玻色子退耦，仅三个大质量自由度有贡献。

The sign of $c_F^{(\text{grav})}$ and $c_F^{(V)}$ is positive, while contributions from scalar and fermion fluctuations have the opposite sign with negative $c_F^{(S)}$ and $c_F^{(F)}$. An overall positive sign of c_F restricts the number of scalars and fermions.

$c_F^{(\text{grav})}$ 和 $c_F^{(V)}$ 的符号为正，而标量涨落和费米子涨落的贡献符号相反，对应 $c_F^{(S)}$ 和 $c_F^{(F)}$ 为负。c_F 的整体正号对标量和费米子的数目给出了限制。

Flow Equation for Kinetial

动力学项流方程

The flow equation for the kinetial K is not yet known reliably. A reliable computation needs to reproduce the property that for the scalar-gravity system one has an enhanced local Weyl symmetry in the limit $\chi \to \infty, K/\xi_\infty \to -6$. After a Weyl scaling to the Einstein frame, the scalar φ appears no longer in the effective action if $Z = 0$ and $\partial U/\partial\varphi = 0$. This reflects the enhanced local symmetry which transmutes the scalar degree of freedom to a pure gauge degree of freedom of this enhanced symmetry. For this limit the Einstein frame can be viewed as removing the gauge degree of freedom of local Weyl symmetry which no longer couples to the physical sector.

动力学项 K 的流方程目前还无法可靠确定。可靠计算需要重现这一性质: 在极限 $\chi \to \infty, K/\xi_\infty \to -6$ 下，标量-引力系统具有增强的局部外尔对称性。经外尔标度变换到爱因斯坦框架后，若满足 $Z = 0$ 和 $\partial U/\partial\varphi = 0$ ，有效作用量中将不再出现标量场 φ 。这反映了增强的局部对称性，它将标量自由度转化为该增强对称性下的纯规范自由度。在此极限下，爱因斯坦框架可视为消去了不再与物理 sector 耦合的局部外尔对称性的规范自由度。

As a result of this enhanced symmetry, the flow of Z should either vanish for $Z = 0$ and $\partial U/\partial\varphi = 0$ or it should diverge. In both cases the value $Z = 0$ cannot be reached. For the case of vanishing flow the fixed point can be approached asymptotically and one expects for $\partial U/\partial\varphi \to 0$ a flow equation of the type

这种增强对称性的结果是，Z 的流在 $Z = 0$ 和 $\partial U/\partial\varphi = 0$ 下要么为零，要么发散。两种情况下都无法达到值 $Z = 0$ 。若流为零，不动点可被渐近趋近，我们预期对于 $\partial U/\partial\varphi \to 0$ 会存在如下形式的流方程

$$k\partial_k Z = -\eta_Z Z \tag{72}$$

where the anomalous dimension η_Z can depend on other couplings and may vanish. In turn, this translates to

其中反常维度 η_Z 可依赖于其他耦合，也可能为零。进而这可转换为

$$k\partial_k\left(\frac{K}{\xi}\right)(\chi \to \infty) = -\eta_Z\left(\frac{K}{\xi} + 6\right), \tag{73}$$

with

其中

$$\xi = \frac{1}{2}\frac{\partial F}{\partial \rho} = \frac{1}{2\chi}\frac{\partial F}{\partial \chi}. \tag{74}$$

It is a somewhat involved task to reproduce the property (73) from the flow equations for K and F . Perturbative one-loop results for these flow equations can be inferred from Ref. [118].

从 K 和 F 的流方程推导出性质 (73) 是一项相当复杂的工作。这些流方程的微扰单圈结果可参见文献 [118]。

The flow equation is formulated at fixed $g_{\mu\nu}$ and χ and not for fixed fields $g'_{\mu\nu}$ and φ in the Einstein frame. The nonlinear field transformation from χ to φ and from $g_{\mu\nu}$ to $g'_{\mu\nu}$ depends on k . The transformation of the flow equation under a change of field variables is well known and results in an additional term [48, 55, 90, 151]

流方程是在固定 $g_{\mu\nu}$ 和 χ 下构造的，而非在爱因斯坦框架中固定场 $g'_{\mu\nu}$ 和 φ 。从 χ 到 φ 、从 $g_{\mu\nu}$ 到 $g'_{\mu\nu}$ 的非线性场变换依赖于 k 。流方程在场变量变换下的变换性质是已知的，会产生一个额外项 [48, 55, 90, 151]

$$\begin{aligned} k\partial_k\Gamma|_{g'_{\mu\nu},\varphi} = k\partial_k\Gamma|_{g_{\mu\nu},\chi} - \int_x \frac{\partial\Gamma}{\partial g'_{\mu\nu}} k\partial_k g'_{\mu\nu}\Big|_{g_{\mu\nu},\chi} \\ -\int_x \frac{\partial\Gamma}{\partial\varphi(x)} k\partial_k\varphi(x)\Big|_{g_{\mu\nu},\chi} \end{aligned} \tag{75}$$

Furthermore, the second functional derivative $\Gamma_k^{(2)}$ has to be translated to functional derivatives with respect to $g'_{\mu\nu}$. (There is a possibility to formulate the IR cutoff term in terms of $g'_{\mu\nu}$ and φ , which yields a formulation where the second functional derivative with respect to these fields appears in the flow equation. Of course, the same cutoff function has to be used for the computation of $k\partial_k U, k\partial_k F$, and $k\partial_k K$.) For the Weyl scaling to the Einstein frame, $g'_{\mu\nu} = \left(F/M^2\right) g_{\mu\nu}$, Eq. (75) reads

此外，二阶泛函导数 $\Gamma_k^{(2)}$ 需要转换为对 $g'_{\mu\nu}$ 的泛函导数。(我们可以将红外截断项表述为 $g'_{\mu\nu}$ 和 φ 的函数，这样流方程中就会出现对这些场的二阶泛函导数。当然，计算 $k\partial_k U, k\partial_k F$ 和 $k\partial_k K$ 时必须使用相同的截断函数。) 对于到爱因斯坦框架的外尔标度变换，$g'_{\mu\nu} = \left(F/M^2\right) g_{\mu\nu}$ ，式 (75) 可写为

$$\begin{aligned} k\partial_k\Gamma|_{g'_{\mu\nu},\varphi} = k\partial_k\Gamma|_{g_{\mu\nu},\chi} + \frac{k^2 c_F}{M^2}\int_x \sqrt{g'}\tilde{T}_E^{\mu\upsilon} g'_{\mu\upsilon} \\ +4M\int_x \frac{\partial\Gamma}{\partial\varphi(x)}, \end{aligned} \tag{76}$$

where the second term involves the trace of the energy momentum tensor $\tilde{T}_E^{\mu\upsilon}$ in the Einstein frame, including here a "gravitational part" according to $\partial\Gamma/\partial g^7_{\mu\nu} = -\left(\sqrt{g'}/2\right)\tilde{T}_E^{\mu\upsilon}$. For solutions of the field equations the additional terms vanish.

其中第二项包含爱因斯坦框架中能量动量张量 $\tilde{T}_E^{\mu\upsilon}$ 的迹，根据 $\partial\Gamma/\partial g^7_{\mu\nu} = -\left(\sqrt{g'}/2\right)\tilde{T}_E^{\mu\upsilon}$ ，这里包含“引力部分”。对于场方程的解，附加项为零。

Equation (76) may permit to compute the flow of the wave function in the Einstein frame and to devise a form of the cutoff function R_k that is compatible with the enhanced local Weyl symmetry. In the Einstein frame one may expect that for small k^2/M^2 the metric fluctuations effectively decouple for the flow of Z . This short discussion demonstrates the work that needs to be done.

式 (76) 或许可以用来计算爱因斯坦框架中波函数的流，并构造出与增强局部外尔对称性相容的截断函数 R_k 形式。在爱因斯坦框架中，我们可以预期，当 k^2/M^2 很小时，度规涨落对 Z 的流的作用有效退耦。这一简短讨论说明了还有许多工作有待完成。

Scaling Solution

标度解

The scaling solution plays a central role for our discussion of cosmology. In this section we therefore investigate the scaling solution for the dimensionless functions $u(\tilde{\rho})$ and $f(\tilde{\rho})$ in some detail. In particular, we discuss the robustness of the important limiting behavior for $\tilde{\rho} \to 0$ and $\tilde{\rho} \to \infty$. The functions $u(\tilde{\rho})$ and $f(\tilde{\rho})$ are sufficient in order to determine the frame-invariant potential $\hat{V}(\tilde{\rho})$ for the scaling solution. For the second frame-invariant function $\hat{K}(\tilde{\rho})$, further computation is required.

标度解在我们的宇宙学讨论中占据核心地位，因此本节我们将详细研究无量纲函数 $u(\tilde{\rho})$ 和 $f(\tilde{\rho})$ 的标度解。我们还将重点讨论 $\tilde{\rho} \to 0$ 和 $\tilde{\rho} \to \infty$ 重要极限行为的鲁棒性。确定标度解的规范不变势 $\hat{V}(\tilde{\rho})$ 仅需用到函数 $u(\tilde{\rho})$ 和 $f(\tilde{\rho})$ ，而第二个规范不变函数 $\hat{K}(\tilde{\rho})$ 则需要进一步计算。

Differential Equations for Scaling Solutions

标度解的微分方程

A scaling solution requires that u and f are only functions of $\tilde{\rho} = \chi^2/\left(2k^2\right)$, solving the flow equation at fixed $\tilde{\rho}$. The flow equations (51) and (65) are evaluated at fixed $\rho = \chi^2/2$

标度解要求 u 和 f 仅为 $\tilde{\rho} = \chi^2/\left(2k^2\right)$ 的函数，在固定 $\tilde{\rho}$ 下求解流方程。流方程 (51) 和 (65) 在固定 $\rho = \chi^2/2$ 处取值

$$k\partial_k u\big|_\rho = -4u + 4c_U,\ k\partial_k f = -2f + 2c_F. \tag{77}$$

The flow equations at fixed $\tilde{\rho}$ are obtained by a variable change

固定 $\tilde{\rho}$ 处的流方程可通过变量变换得到

$$k\partial_k u\big|_{\tilde{\rho}} = k\partial_k u\big|_\rho - \frac{\partial u}{\partial \tilde{\rho}} k\partial_k \tilde{\rho}\big|_\rho \tag{78}$$

where

其中

$$k\partial_k\tilde{\rho} \mid \rho = -2\tilde{\rho} \tag{79}$$

and similar for f . In terms of the dimensionless scalar fields, Eq. (77) transforms to

对 f 有类似形式。用无量纲标量场表示，式 (77) 可变换为

$$\left(k\partial_k - 2\tilde{\rho}\partial_{\tilde{\rho}}\right)u = 4\left(c_U - u\right)$$

$$\left(k\partial_k - 2\tilde{\rho}\partial_{\tilde{\rho}}\right)f = 2\left(c_F - f\right). \tag{80}$$

In our truncation the quantities c_U and c_F are functions of $\tilde{\rho}$, involving u, f , and derivatives thereof. Equation (80) constitutes a closed system of differential equations.

在我们的截断中，量 c_U 和 c_F 是 $\tilde{\rho}$ 的函数，包含 u, f 及其导数。式 (80) 构成了一个封闭的微分方程组。

The scaling solution solves Eq. (80) with $k\partial_k u|_{\tilde{\rho}} = 0, k\partial_k f|_{\tilde{\rho}} = 0$. We infer the differential equations that scaling solutions have to obey,

标度解在满足 $k\partial_k u|_{\tilde{\rho}} = 0, k\partial_k f|_{\tilde{\rho}} = 0$ 的条件下求解式 (80)。我们推导出标度解必须满足的微分方程,

$$\tilde{\rho}\partial_{\tilde{\rho}}u = 2\left(u - c_U\right), \ \tilde{\rho}\partial_{\tilde{\rho}}f = f - c_F. \tag{81}$$

These are central equations for this work. The properties of the scaling functions $u\left(\tilde{\rho}\right)$ and $f\left(\tilde{\rho}\right)$, and in particular their behavior in the limits $\tilde{\rho} \to 0$ and $\tilde{\rho} \to \infty$, follow from the solutions of these differential equations. As a boundary condition we require that for $\tilde{\rho} \to 0$ both u and f reach finite values:

这些是本文的核心方程。标度函数 $u\left(\tilde{\rho}\right)$ 和 $f\left(\tilde{\rho}\right)$ 的性质，特别是它们在极限 $\tilde{\rho} \to 0$ 和 $\tilde{\rho} \to \infty$ 下的行为，可由这些微分方程的解得到。作为边界条件，我们要求当 $\tilde{\rho} \to 0$ 时，u 和 f 都收敛到有限值:

$$u\left(\tilde{\rho} = 0\right) = u_0 = c_U\left(\tilde{\rho} = 0\right), \ f\left(\tilde{\rho} = 0\right) = f_0 = c_F\left(\tilde{\rho} = 0\right). \tag{82}$$

In turn, this requires finite values of c_U and c_F for $\tilde{\rho} \to 0$. Together with the requirement that u and f (and therefore also c_U and c_F) should remain finite for any finite value of $\tilde{\rho}$, these conditions severely restrict the possible scaling solutions.

相应地，这要求当 $\tilde{\rho} \to 0$ 时 c_U 和 c_F 取有限值。再结合对于任意有限的 $\tilde{\rho}$ ，u 和 f (因此也包括 c_U 和 c_F) 都保持有限的要求，这些条件对可能的标度解作出了严格限制。

Keeping in mind the corrections discussed above, we approximate the flow generators for u by

考虑到上文讨论的修正，我们将 u 的流生成元近似为

$$c_U = \frac{1}{96\pi^2}\left(1 - \frac{1}{4}\tilde{\rho}\partial_{\tilde{\rho}} \ln f\right)\left(\frac{5}{1-v} + \frac{1}{1-v/4}\right) + \frac{\mathcal{N}_U}{128\pi^2}, \tag{83}$$

with

其中

$$\mathcal{N}_U = \bar{N}_S + 2\bar{N}_V - 2\bar{N}_F - 4. \tag{84}$$

The generator c_F will be approximated by

生成元 c_F 将被近似为

$$c_F = \frac{\left(1 - \frac{1}{3}\tilde{\rho}\partial_{\tilde{\rho}} \ln f\right)}{64\pi^2}\left(\frac{25}{1-v} - \frac{8}{9\,(1-v/4)}\right) + \frac{\mathcal{N}_F}{96\pi^2}, \tag{85}$$

where

其中

$$\mathcal{N}_F = -\bar{N}_S - \bar{N}_F + 4\bar{N}_V' + \frac{17}{2}. \tag{86}$$

Here we employ for the scaling solution $\eta_g = 2\tilde{\rho}\partial_{\tilde{\rho}} \ln f$ and we simplify its slightly different role for the graviton and scalar metric fluctuations. We have omitted the term $c_F^{(\xi)}$ in Eq. (67). The main corrections are presumably due to the omission of mixing between the scalar metric fluctuations and additional scalars. Since the contribution of the scalar metric fluctuations is substantially smaller than the one from the graviton fluctuations (”graviton dominance”), the main characteristics should be well described by the approximation (83)-(86).

我们此处针对标度解 $\eta_g = 2\tilde{\rho}\partial_{\tilde{\rho}} \ln f$ ，简化了其在引力子和标度度量涨落中略有差异的作用。我们省略了式 (67) 中的项 $c_F^{(\xi)}$ 。主要修正大概来自于对标量度量涨落与额外标量之间混合项的省略。由于标量度量涨落的贡献远小于引力子涨落的贡献 (“引力子主导”)，近似式 (83)-(86) 应当可以很好地描述主要特征。

The range of validity of the flow equations is restricted to $v < 1$. For $v = 1$ the graviton propagator is divergent even in presence of the IR cutoff. One finds that $v = 1$ constitutes a barrier in the flow that is not crossed [140]. This is related to general convexity properties of the scale-dependent effective action [120].

流方程的适用范围被限制为 $v < 1$ 。当 $v = 1$ 时，即使存在红外截断，引力子传播子仍是发散的。研究发现 $v = 1$ 构成了流中一道无法跨越的势垒 [140]，这与依赖能标有效作用量的一般凸性性质相关 [120]。

Limiting Behavior of Scaling Solutions

标度解的极限行为

For large $\tilde{\rho} \to \infty$ one finds a simple solution

对于大的 $\tilde{\rho} \to \infty$ ，可以得到一个简单解

$$f(\tilde{\rho}) = 2\xi_\infty \tilde{\rho}, \; F = \xi_\infty \chi^2. \tag{87}$$

Indeed, for finite c_F a term $2\xi_\infty \tilde{\rho}$ dominates the r.h.s. of Eq. (81) for $\xi_\infty \neq 0$. A similar solution $u \sim \tilde{\rho}^2$ is not possible since it would lead to divergent v for positive u , or to negative divergent u , which is forbidden by convexity properties in the scalar sector. What remains is a constant value

事实上，对于有限 c_F ， 当 $\xi_\infty \neq 0$ 时，项 $2\xi_\infty \tilde{\rho}$ 主导了式 (81) 的右端。类似解 $u \sim \tilde{\rho}^2$ 不可能存在，因为它会在 u 为正时导致发散的 v ，或是导致负的发散 u ，这被标量区的凸性性质所禁止。剩余可能的是一个常数取值

$$u(\tilde{\rho} \to \infty) = u_\infty = \frac{1}{128\pi^2} \left(2 + \bar{N}_S + 2\bar{N}_V - 2\bar{N}_F\right). \tag{88}$$

Here we note that for $f \sim \tilde{\rho}, u \to u_\infty$, one has $v \to 0, \eta_g \to 2$. At this point we have a whole family of possible scaling solutions parameterized by ξ_∞ . Not all of them may correspond to true scaling solutions that remain valid for the whole range $0 \leq \tilde{\rho} < \infty$. For $\xi_\infty \neq 0$ we can expand the flow equation in inverse powers of $\tilde{\rho}$, with fixed coefficients for given ξ_∞ [64]. The case $\xi_\infty = 0$ is special. It corresponds to the constant scaling solution (generalized Reuter fixed point) for which u and f are independent of $\tilde{\rho}$ and take the same value as for $\tilde{\rho} = 0$.

这里我们注意到，对于 $f \sim \tilde{\rho}, u \to u_\infty$ ， 有 $v \to 0, \eta_g \to 2$ 。至此我们得到了一整族以 ξ_∞ 为参数的可能标度解。并非所有解都对应在整个区间 $0 \leq \tilde{\rho} < \infty$ 上都成立的真标度解。对于 $\xi_\infty \neq 0$ ， 我们可以将流方程按 $\tilde{\rho}$ 的负幂次展开，给定 ξ_∞ 时系数为常数 [64]。$\xi_\infty = 0$ 的情况是特殊的，它对应常数标度解 (推广的 Reuter 不动点)，对于该解 u 和 f 都与 $\tilde{\rho}$ 无关，且取值和 $\tilde{\rho} = 0$ 的情况相同。

In the limit $\tilde{\rho} \to 0$ both u and f approach constants

在 $\tilde{\rho} \to 0$ 极限下，u 和 f 都趋近于常数

$$u(\tilde{\rho} \to 0) = u_0 = c_U(0)$$

$$= \frac{1}{128\pi^2} \left[\frac{4}{3}\left(\frac{5}{1 - v_0} + \frac{1}{1 - v_0/4}\right)\right.$$

$$\left. -4 + \bar{N}_S + 2\bar{N}_V - 2\bar{N}_F\right], \tag{89}$$

and

且

$$f(\tilde{\rho} \to 0) = f_0 = c_F(0)$$

$$= \frac{1}{96\pi^2}\left[\frac{3}{2}\left(\frac{25}{1-v_0} - \frac{8}{9(1-v_0/4)}\right)\right.$$

$$\left. + \frac{17}{2} - \bar{N}_S - \bar{N}_F + 4\bar{N}_V'\right]. \tag{90}$$

Here we employ $v_0 = u_0/(2f_0)$ and $\eta_g = 0$. Inserting this value, Eqs. (89) and (90) are two coupled nonlinear equations for u_0 and f_0 . A discussion of the possible solutions with $f_0 > 0$ in dependence on the numbers of effectively massless particles can be found in Ref. [154], or, for somewhat different flow equations, in Ref. [21, 33, 40, 96].

此处我们使用 $v_0 = u_0/(2f_0)$ 和 $\eta_g = 0$ 。代入该取值后，式 (89) 和式 (90) 成为关于 u_0 和 f_0 的两个耦合非线性方程。关于 $f_0 > 0$ 条件下可能解随有效无质量粒子数的变化讨论，可以参考文献 [154]；针对不同形式的流方程的相关讨论可见参考文献 [21, 33, 40, 96]。

Scaling Solution for Potential

势的标度解

For scaling solutions with $\tilde{\rho}$ -dependent masses or $\xi_\infty \neq 0$, the effective numbers of particles are different for $\tilde{\rho} \to 0$ and $\tilde{\rho} \to \infty$. As $\tilde{\rho}$ increases, more and more particles decouple from the flow since their masses become larger than k . We may write

对于具有 $\tilde{\rho}$ 依赖质量或 $\xi_\infty \neq 0$ 的标度解，有效粒子数在 $\tilde{\rho} \to 0$ 和 $\tilde{\rho} \to \infty$ 中并不相同。随着 $\tilde{\rho}$ 增大，越来越多粒子的质量大于 k ，因而从流中退耦，我们可以写出

$$c_U = \sum_j c_U^{(j)} + c_U^{\text{grav}} - \frac{N_V^{(0)}}{128\pi^2}, \tag{91}$$

with

其中

$$c_U^{(j)} = \frac{N_j^{(u)}}{128\pi^2\left(1 + \tau_j\tilde{\rho}\right)}, \tag{92}$$

the contribution of particles with mass given by $m_j^2 = \tau_j\chi^2/2, \tilde{m}_j^2 = \tau_j\tilde{\rho}$. Here $N_j^{(u)} = N_{S,j} + 3N_{V,j} - 2N_{F,j}$ involves the appropriate combination of scalars, gauge bosons, and Majorana fermions with mass m_j . (This effective number may include contributions from the anomalous dimension.) The number $N_V^{(0)}$ denotes the number of massless gauge bosons and the last term in Eq. (91) arises from the measured term for the massless gauge bosons. For the massless gauge bosons this subtracts one unit from $3N_{V,j}$, such that only $2N_V^{(0)}$ massless degrees of freedom contribute. The contributions of the $N_V - N_V^{(0)}$ Goldstone bosons are approximated here as massless even away from the potential minimum. As we have discussed before, their contribution is canceled by the measured terms for the $N_V - N_V^{(0)}$ massive gauge bosons. In consequence, neither the Goldstone boson fluctuations not the measured terms for the massive gauge bosons appear in $\sum_j c_U^{(j)}$ in Eq. (91).

质量由 $m_j^2 = \tau_j \chi^2/2, \tilde{m}_j^2 = \tau_j \tilde{\rho}$ 给出的粒子的贡献。此处 $N_j^{(u)} = N_{S,j} + 3N_{V,j} - 2N_{F,j}$ 包含标量、规范玻色子和质量为 m_j 的马约拉纳费米子的恰当组合。(该有效数可包含反常维数的贡献。) 数 $N_V^{(0)}$ 表示无质量规范玻色子的数量，式 (91) 的最后一项来自无质量规范玻色子的测量项。对于无质量规范玻色子，该项从 $3N_{V,j}$ 中减去一个单位，使得只有 $2N_V^{(0)}$ 个无质量自由度贡献。此处将 $N_V - N_V^{(0)}$ 个戈德斯通玻色子的贡献近似为无质量，即使在远离势极小值处也是如此。正如我们之前讨论的，它们的贡献会被 $N_V - N_V^{(0)}$ 个有质量规范玻色子的测量项抵消。因此，戈德斯通玻色子涨落和有质量规范玻色子的测量项都不会出现在式 (91) 的 $\sum_j c_U^{(j)}$ 中。

The scaling solution of the differential equation (81) yields

微分方程 (81) 的标度解给出

$$u = \sum_j u_j + u_{\text{grav}} \tag{93}$$

where

其中

$$u_j = \frac{N_j^{(u)}}{128\pi^2} t_u\left(\tau_j \tilde{\rho}\right) \tag{94}$$

involves the threshold function $t_u\left(\tau_j \tilde{\rho}\right)$. This threshold function,

包含阈函数 $t_u\left(\tau_j \tilde{\rho}\right)$。这个阈函数

$$t_u\left(\tau_j \tilde{\rho}\right) = 1 - 2\tau_j \tilde{\rho} - 2\left(\tau_j \tilde{\rho}\right)^2 \ln\left(\frac{\tau_j \tilde{\rho}}{1 + \tau_j \tilde{\rho}}\right), \tag{95}$$

interpolates between the limits

在两个极限之间插值

$$t_u\left(\tau_j \tilde{\rho} \ll 1\right) = 1 - 2\tau_j \tilde{\rho},\ t_u\left(\tau_j \tilde{\rho} \gg 1\right) = \frac{2}{3\tau_j \tilde{\rho}}, \tag{96}$$

and obeys

且满足

$$y \frac{\partial t_u(y)}{\partial y} = 2t_u(y) - \frac{2}{1+y}. \tag{97}$$

The metric contribution obeys

度规贡献满足

$$\left(\tilde{\rho}\partial_{\tilde{\rho}} - 2\right) u_{\text{grav}} = -2c_{U,\,\text{grav}}$$

$$c_{U,\,\text{grav}} = \frac{1}{96\pi^2}\left(1 - \frac{1}{4}\tilde{\rho}\partial_{\tilde{\rho}} f\right)\left(\frac{5}{1-v} + \frac{1}{1-v/4}\right) - \frac{1}{32\pi^2}. \tag{98}$$

For large $f \approx 2\xi\tilde{\rho}$ one has $v \to 0$ and u_{grav} approaches a constant

对于大 $f \approx 2\xi\tilde{\rho}$ ，有 $v \to 0$ ，且 u_{grav} 趋近于常数

$$u_{\text{grav}} = \frac{1}{64\pi^2} + O\left(\frac{1}{f}\right). \tag{99}$$

Together with $N_V = 1$ for the photon and $N_S = 1$ for the scalar field, this yields Eq. (14).

结合光子的 $N_V = 1$ 和标量场的 $N_S = 1$ ，即可得到式 (14)。

Neutrinos, Standard Model, and Grand Unification

中微子、标准模型与大统一理论

Neutrinos are much lighter than the other fermions by virtue of the seesaw mechanism [54, 76, 80, 81, 156] and play an interesting role. For k smaller than the neutrino masses, only the metric fluctuations, the photon, and the cosmon contribute to the flow leading to $2 + \bar{N}_S + 2\bar{N}_V = 5$ and therefore positive u_0 . Once k exceeds the heaviest of the neutrino masses, one finds $2 + \bar{N}_S + 2\bar{N}_V - 2\bar{N}_F = -1$ and therefore negative c_U . If we simplify to three equal neutrino masses, $m_v = h_v M$ in the Einstein frame, the scaling equation for u in the region where the "neutrino threshold" is cross read $\left(\tilde{m}_v^2 = 2h_v^2\tilde{\rho}, \tau_v = 2h_v^2\right)$:

中微子因跷跷板机制 [54, 76, 80, 81, 156] 比其他费米子轻得多，并扮演着值得关注的角色。当 k 小于中微子质量时，只有度规涨落、光子和宇宙子对流向产生贡献，最终得到 $2 + \bar{N}_S + 2\bar{N}_V = 5$ ，因此 u_0 为正。一旦 k 超过最重中微子的质量，就会得到 $2 + \bar{N}_S + 2\bar{N}_V - 2\bar{N}_F = -1$ ，因此 c_U 为负。如果我们简化为三种质量相等的中微子，在爱因斯坦框架下 $m_v = h_v M$ ，当穿过“中微子阈值”区域时，u 满足的标度方程为 $\left(\tilde{m}_v^2 = 2h_v^2\tilde{\rho}, \tau_v = 2h_v^2\right)$:

$$\tilde{\rho}\partial_{\tilde{\rho}} u = 2u - \frac{1}{64\pi^2}\left(5 - \frac{6}{1 + 2h_v^2\tilde{\rho}}\right). \tag{100}$$

Starting from positive $u \approx u_0$ for $\tilde{\rho} \gg h_v^{-2}$ and decreasing $\tilde{\rho}$, the positive r.h.s. of Eq. (100) drives u to smaller values. For $h_v^2\tilde{\rho} \gg 1$ this results in

从 $\tilde{\rho} \gg h_v^{-2}$ 对应正的 $u \approx u_0$ 、且 $\tilde{\rho}$ 递减开始，式 (100) 右侧为正，会将 u 推向更小的值。对于 $h_v^2\tilde{\rho} \gg 1$ ，结果为

$$u = \frac{5}{128\pi^2} - \frac{1}{64\pi^2 h_v^2 \tilde{\rho}}, \tag{101}$$

while for $h_v^2\tilde{\rho} \ll 1$ a new constant scaling solution with negative u is approached from above

而对于 $h_v^2\tilde{\rho} \ll 1$ ，会从上方趋近于一个 u 为负的新常数标度解

$$u = -\frac{1}{128\pi^2} + \frac{3}{16\pi^2} h_v^2\tilde{\rho}. \tag{102}$$

We observe that c_U turns negative for $h_v^2\tilde{\rho} = 1/10$ or $k^2 = m_v^2/5$. Associating roughly $u_0 k^4$ with the present dark energy density $\approx \left(2 \cdot 10^{-3}\text{eV}\right)^4$ yields k in the region of 10^{-2}eV , rather close to the experimental lower limit for the largest neutrino mass. As $\tilde{\rho}$ decreases further u gets more negative due to the large number of fermions in the standard model.

我们可以看到，c_U 在 $h_v^2\tilde{\rho} = 1/10$ 或 $k^2 = m_v^2/5$ 的情况下变为负值。如果将当前暗能量密度 $\approx \left(2 \cdot 10^{-3}\text{eV}\right)^4$ 粗略对应为 $u_0 k^4$ ，可得 k 位于 10^{-2}eV 的区间内，与最大中微子质量的实验下限非常接近。随着 $\tilde{\rho}$ 进一步减小，由于标准模型中存在大量费米子，u 会变得更负。

For grand unified theories c_U is typically positive above the unification scale due to the large number of gauge bosons and scalars. With $u_0 > 0$, negative u between the unification scale and neutrino mass and positive u_∞ , the Einstein potential $\hat{V}$ has a rich structure. Approximating $f(\tilde{\rho}) = f_0 + 2\xi_\infty\tilde{\rho}$ it approaches a constant for $\tilde{\rho} \to 0, \varphi \to -\infty$, decays exponentially for increasing φ once $2\xi_\infty\tilde{\rho} > f_0$, turns negative in the vicinity of the unification scale for $\tilde{\rho} \approx 10^4$, has a minimum for somewhat larger φ (say $\tilde{\rho} \approx 10^5$) at negative values, turns positive again for k below the neutrino mass $\left(\tilde{\rho} \approx h_v^{-2}\right)$, and finally decays exponentially for $\varphi \to \infty$. This may lead to interesting features in the cosmological evolution if the solution of the flow equations follows the scaling solution up to $\tilde{\rho} \approx 10^{116}$, corresponding to $u/\left(f^2\right) = u/(2\xi_\infty\tilde{\rho})^2 \approx 10^{-120}$.

对于大统一理论，由于规范玻色子和标量粒子数量众多，c_U 在统一能标之上通常为正。在 $u_0 > 0$ 的条件下，u 在统一能标和中微子质量之间为负，u_∞ 为正，因此爱因斯坦势 $\hat{V}$ 具有丰富的结构。对 $f(\tilde{\rho}) = f_0 + 2\xi_\infty\tilde{\rho}$ 做近似后，它在 $\tilde{\rho} \to 0, \varphi \to -\infty$ 时趋近于常数；当 $2\xi_\infty\tilde{\rho} > f_0$ 满足后，它随 φ 增大指数衰减；对于 $\tilde{\rho} \approx 10^4$ ，它在统一能标附近变为负值；它在稍大的 φ 处 (例如 $\tilde{\rho} \approx 10^5$) 取负值时存在一个极小值；当 k 低于中微子质量 $\left(\tilde{\rho} \approx h_v^{-2}\right)$ 时，它再次变为正值，最终在 $\varphi \to \infty$ 时指数衰减。如果流方程的解在达到 $\tilde{\rho} \approx 10^{116}$ (对应 $u/\left(f^2\right) = u/(2\xi_\infty\tilde{\rho})^2 \approx 10^{-120}$.) 前都遵循标度解，这可能会为宇宙演化带来有趣的特征

Scaling Solution for the Curvature Coefficient and Kinetial

曲率系数与动理学量的标度解

For a constant ξ we define

对于常数 ξ 我们定义

$$f(\tilde{\rho}) = \widetilde{f}(\tilde{\rho}) + 2\xi\tilde{\rho}. \tag{103}$$

Then the scaling solution for $\tilde{f}$ obeys

则 $\tilde{f}$ 的标度解满足

$$\tilde{\rho}\partial_{\tilde{\rho}}\tilde{f} = \tilde{f} - c_F. \tag{104}$$

We identify ξ with ξ_∞ ,

我们将 ξ 与 ξ_∞ 等同,

$$\xi = \lim_{\tilde{\rho}\to\infty} \frac{f(\tilde{\rho})}{2\tilde{\rho}}, \lim_{\tilde{\rho}\to\infty} \frac{\tilde{f}(\tilde{\rho})}{2\tilde{\rho}} = 0. \tag{105}$$

The scaling solution for $\tilde{f}$ is similar to the one for u , with a finite value $\tilde{f}(\tilde{\rho} \to \infty) = \tilde{f}_\infty$. Similar to the flow equation for u we approximate

$\tilde{f}$ 的标度解与 u 的标度解类似，具有有限值 $\tilde{f}(\tilde{\rho} \to \infty) = \tilde{f}_\infty$ 。与 u 的流方程类似，我们近似为

$$c_F = \sum_j \frac{N_j^{(f)}}{96\pi^2\left(1+\tau_j\tilde{\rho}\right)} + c_F^{(\mathrm{grav})} + \frac{N_V^{(0)}}{96\pi^2}. \tag{106}$$

The numbers $N_j^{(f)} = -N_S^{(j)} - N_F^{(j)} + 3N_V^{(j)}$ involve the corresponding numbers of scalars, Majorana fermions, and gauge bosons with dimensionless squared mass $\tilde{m}_j^2 = \tau_j\tilde{\rho}$. For massless gauge bosons the addition of the measured term enhances $2N_V^{(0)}$ to $4N_V^{(0)}$. Again, the Goldstone bosons corresponding to the massive gauge bosons do not contribute.

数目 $N_j^{(f)} = -N_S^{(j)} - N_F^{(j)} + 3N_V^{(j)}$ 包含了标量、马约拉纳费米子以及无量纲平方质量为 $\tilde{m}_j^2 = \tau_j\tilde{\rho}$ 的规范玻色子的对应数目。对于无质量规范玻色子，添加测量项后将 $2N_V^{(0)}$ 提升为 $4N_V^{(0)}$ 。此外，对应有质量规范玻色子的戈德斯通玻色子没有贡献。

In this approximation one finds the solution

在该近似下可得解

$$\tilde{f} = \sum_j \tilde{f}_j + \tilde{f}_{\mathrm{grav}}, \tag{107}$$

where

其中

$$\tilde{f}_j = \frac{N_j^{(f)}}{96\pi^2} t_f\left(\tau_j\tilde{\rho}\right), \tag{108}$$

with threshold function

阈函数为

$$t_f(y) = 1 + y \ln\left(\frac{y}{1+y}\right) \tag{109}$$

obeying

满足

$$y\partial_y t_f = t_f - \frac{1}{1+y}. \tag{110}$$

We may write

我们可以写作

$$\widetilde{f}(\rho) = c_F(\tilde{\rho}) + \Delta_\rho(\tilde{\rho}), \tag{111}$$

where $\Delta_\rho(\tilde{\rho})$ differs from zero only in the threshold regions where the precise $\tilde{\rho}$ -dependence of $\widetilde{f}(\tilde{\rho})$ differs from the one for $c_F(\tilde{\rho})$. For large $\tilde{\rho}$ the detailed form of $\widetilde{f}$ becomes unimportant, $\widetilde{f}(\tilde{\rho})$ being subleading as compared to $2\xi\tilde{\rho}$.

其中仅当阈区中 $\widetilde{f}(\tilde{\rho})$ 对 $\tilde{\rho}$ 的精确依赖关系与 $c_F(\tilde{\rho})$ 的依赖关系不同时，$\Delta_\rho(\tilde{\rho})$ 才不为零。对于大 $\tilde{\rho}$ ，$\widetilde{f}$ 的具体形式变得无关紧要，因为 $\widetilde{f}(\tilde{\rho})$ 与 $2\xi\tilde{\rho}$ 相比是次 leading 项。

In our approximation the coupling ξ appears only in the metric contribution to the flow equation through v and $\tilde{\rho}\partial_{\tilde{\rho}} \ln f = 1 - c_F/\left(2\xi\tilde{\rho} + \widetilde{f}\right)$. For large $\tilde{\rho}$ the metric contributions are suppressed by $(\xi\tilde{\rho})^{-1}$. Only the effectively massless particles contribute in this range. The Weyl transformation to the Einstein frame reveals, however, that for a given normalization of K , say $K_\infty = \pm 1$, the value of ξ enters the wave function renormalization Z . The so far neglected mixing of kinetic terms for χ and the scalar metric fluctuations are important for understanding the precise role of ξ in the region for large χ .

在我们的近似中，耦合 ξ 仅通过 v 和 $\tilde{\rho}\partial_{\tilde{\rho}} \ln f = 1 - c_F/\left(2\xi\tilde{\rho} + \widetilde{f}\right)$ 出现在流方程的度规贡献中。对于大 $\tilde{\rho}$ ，度规贡献被 $(\xi\tilde{\rho})^{-1}$ 压低。该范围内只有有效无质量粒子有贡献。然而，对爱因斯坦框架作外尔变换后可以发现，对于给定归一化的 K ，例如 $K_\infty = \pm 1$ ，ξ 的值会进入波函数重整化 Z 。目前被忽略的 χ 动能项混合与标量度规涨落，对理解大 χ 区域中 ξ 的精确作用十分重要。

We can also employ Eq. (81) for the scaling solution in order to express the wave function renormalization as

我们也可以利用标度解的式 (81) 将波函数重整化表示为

$$Z = \frac{1}{8}\left[\frac{K\tilde{\rho}}{f} + 3\left(1 - \frac{c_F}{f}\right)^2\right]. \tag{112}$$

For large $\tilde{\rho}$ we can neglect c_F/f and recover Eq. (27). As long as the flow equation and the scaling form for the kinetial $K(\tilde{\rho})$ are not computed, we can only discuss possible forms which lead to realistic cosmology.

The fact that the scaling solution is known for only one of the two scale-invariant functions (7) relevant for cosmology, namely, $\hat{V}(\tilde{\rho})$, clearly limits the predictive power. In principle, the form of $K(\tilde{\rho})$ matters for a precise determination of $\hat{V}(\tilde{\rho})$. This effect is small, however, since the relative contribution of the scalar singlet fluctuations to the flow of U and F is small.

对于大 $\tilde{\rho}$ 我们可以忽略 c_F/f ，得到式 (27)。只要动理学量 $K(\tilde{\rho})$ 的流方程和标度形式尚未被计算，我们就只能讨论能够得到现实宇宙学的可能形式。目前仅已知对宇宙学 relevant 的两个尺度不变函数 (7) 中的一个即 $\hat{V}(\tilde{\rho})$ 存在标度解，这显然限制了预言能力。原则上，$K(\tilde{\rho})$ 的形式对精确确定 $\hat{V}(\tilde{\rho})$ 是重要的。不过该效应很小，因为标量单态涨落对 U 和 F 流的相对贡献很小。

Robustness of Limits of the Scaling Solution

标度解极限的鲁棒性

One may ask how robust the results are for the limiting behavior of the scaling solution for $\tilde{\rho} \to 0$ and $\tilde{\rho} \to \infty$. As we have argued in section "Quantum Gravity" the result (13), (15) entails important aspects for the understanding of the overall evolution of the universe. There is actually only a rather limited set of properties that enter this result. First, for any flow equation which ensures a proper decoupling of heavy particles, the generic behavior

人们或许会问，$\tilde{\rho} \to 0$ 和 $\tilde{\rho} \to \infty$ 对应的标度解极限行为的结果鲁棒性如何。正如我们在“量子引力”小节中所说，结果 (13)、(15) 对理解宇宙整体演化具有重要意义。实际上，只有相当少的一组性质会影响该结果。首先，对任何能保证重粒子恰当退耦的流方程，其一般行为

$$k\partial_k U \sim k^4,\ k\partial_k F \sim k^2 \tag{113}$$

is dictated by dimensions. With effective particle masses vanishing (except for threshold regions), the scale k is the only scale relevant for the flow. This assures constant values u_0, f_0 for $\tilde{\rho} \to 0$ unless one has a highly nonanalytic behavior with u or f increasing $\sim \tilde{\rho}^{-1}$ or faster.

由量纲决定。有效粒子质量 (除阈值区域外) 趋近于零，因此标度 k 是唯一对流动相关的标度。这就保证了除非出现 u 或 f 以不低于 $\sim \tilde{\rho}^{-1}$ 的速度增长的高度非解析行为，否则 $\tilde{\rho} \to 0$ 的 u_0, f_0 将保持为常数。

Second, there is no good reason why the non-minimal gravitational coupling of the scalar field ξ should be zero for $\tilde{\rho} \to \infty$. For positive ξ_∞ one finds $F \sim \xi_\infty \chi^2$ for large χ . Thus, for $\tilde{\rho} \to \infty$ the fluctuations of the metric become negligible as usually assumed for $k^2 < F$. (This assumes a proper diagonalization of the kinetic term in the scalar sector as realized in the Einstein frame.) An exception is a constant contribution to u and f given by the gravitational contribution to c_U and c_F . Together with other massless particles, this leads to nonzero c_U and c_F for $\tilde{\rho} \to \infty$. The constant c_F becomes irrelevant for $f \sim 2\xi_\infty \tilde{\rho}$.

其次，没有充分理由说明标量场 ξ 的非最小引力耦合在 $\tilde{\rho} \to \infty$ 处应当为零。当 ξ_∞ 为正时，大 χ 下可得到 $F \sim \xi_\infty \chi^2$ 。因此，在 $\tilde{\rho} \to \infty$ 处度规涨落变得可以忽略，这和 $k^2 < F$ 下的通常假设一致。(这假设标量部门的动能项已经像爱因斯坦框架中实现的那样完成了恰当对角化。) 一个例外是引力对 c_U 和 c_F 的贡献会给 u 和 f 带来常数贡献。结合其他无质量粒子，这会使得 $\tilde{\rho} \to \infty$ 处的 c_U 和 c_F 非零，而常数 c_F 对 $f \sim 2\xi_\infty \tilde{\rho}$ 无关紧要。

Third, for the flow of U for $k^2 \ll m_v^2$, only the effectively massless particles below the neutrino mass scale contribute. These are the photon, the metric fluctuations, and the cosmon, unless one extends the standard model to include additional massless particles. The expression for c_U becomes rather simple, with a positive sign for bosons and only counting the number of propagating degrees of freedom. This coincides with simple estimates of the ground state energy in the Hamiltonian formalism. One infers $c_U > 0$ for $\tilde{\rho} \to \infty$.

第三，对 $k^2 \ll m_v^2$ 下 U 的流动，只有中微子质量标度以下的有效无质量粒子会产生贡献。除非扩展标准模型纳入额外无质量粒子，这些粒子就是光子、度规涨落和宇宙子。c_U 的表达式变得相当简单，玻色子对应正号，仅计数传播自由度的数量，这与哈密顿形式中基态能量的简单估计一致，由此可以推得 $\tilde{\rho} \to \infty$ 对应的 $c_U > 0$ 。

Fourth, an increase of $u \sim \tilde{\rho}^2$ is not allowed due to stability conditions. This leaves for u only the possibility $u(\tilde{\rho} \to \infty) = u_\infty$. A possible exception could only be a value of v very close to one which invalidates the decoupling of the graviton fluctuations for $\partial u/\partial \tilde{\rho}$ with $u(\tilde{\rho} \to \infty) = 4\xi_\infty \tilde{\rho}$ [140]. We will not pursue this possibility here further since it seems not very likely that a full consistent scaling solution can be obtained for this extreme behavior.

第四，稳定性条件不允许 $u \sim \tilde{\rho}^2$ 增大，因此对 u 而言只剩下 $u(\tilde{\rho} \to \infty) = u_\infty$ 这一种可能。唯一可能的例外是 v 的值非常接近 1，这会使得 $\partial u/\partial \tilde{\rho}$ 在 $u(\tilde{\rho} \to \infty) = 4\xi_\infty \tilde{\rho}$ 时引力子涨落的退耦失效 [140]。我们在此不再进一步讨论这种可能性，因为针对这种极端行为得到一个完全自洽的标度解的可能性似乎很低。

Fifth, the metric fluctuations no longer contribute to $\partial c_U/\partial \tilde{\rho}$ of $\partial c_F/\partial \tilde{\rho}$ for $\xi_\infty \tilde{\rho} \gg 1$. With $v = 2u/f$,

第五，在 $\xi_\infty \tilde{\rho} \gg 1$ 处，度规涨落不再对 $\partial c_F/\partial \tilde{\rho}$ 的 $\partial c_U/\partial \tilde{\rho}$ 产生贡献。在 $v = 2u/f$ 的条件下，

$$\frac{\partial v}{\partial \tilde{\rho}} = \frac{2}{f}\frac{\partial u}{\partial \tilde{\rho}} - \frac{2u}{f^2}\frac{\partial f}{\partial \tilde{\rho}} = -\frac{v}{\tilde{\rho}}, \tag{114}$$

$$\tilde{\rho}\frac{\partial c_U^{(\text{grav})}}{\partial \tilde{\rho}} = -v\frac{\partial c_U^{(\text{grav})}}{\partial v} = -\frac{\left(5 + \frac{1}{4}\right)v}{128\pi^2}. \tag{115}$$

---

one obtains expressions which vanish for $v \to 0$ as

可以得到在 $v \to 0$ 处为零的表达式

In this region only the nongravitational particles contribute to the field dependence of u or f .

该区域中只有非引力粒子会对 u 或 f 的场依赖产生贡献。

Sixth, the flow of the particle physics couplings all obtain from ρ -derivatives of the flow generator for U or F . With gravity decoupled the flow of small couplings follows the perturbative β -functions. Once $\tilde{\rho}$ is large enough such that only the particles of the standard model contribute, one has a good control of the differential equation defining the scaling solution. The fact that the perturbative β -functions are reproduced by simply taking derivatives of the flow equation for the effective potential increases the confidence in the validity of this equation.

第六，粒子物理耦合的流全都由 ρ 对 U 或 F 流生成元求导得到。引力退耦后，小耦合的流遵循微扰 β 函数。一旦 $\tilde{\rho}$ 足够大，使得只有标准模型粒子产生贡献，我们就能很好地控制定义标度解的微分方程。有效势流方程简单求导即可得到微扰 β 函数，这一点增加了我们对该方程有效性的信心。

Taking things together the results for the limiting behavior of the scaling solution seem to be rather robust. The limiting behavior for $\tilde{\rho} \to \infty$ only depends on the massless particles. Changes of the constant $c_U(\tilde{\rho} \to \infty)$ due to the use of different versions of the flow equations or different forms of the infrared cutoff function can be absorbed by a rescaling of k , or a shift of the definition of φ in the Einstein frame. The fact that $u(\tilde{\rho} \to 0)$ and $f(\tilde{\rho} \to 0)$ take constant values seems to be very general. The precise values u_0 and f_0 typically depend on the unknown particle content in the ultraviolet limit, as well as on the precise implementation of flow equations and cutoff functions. One expects that a large class of models leads to positive u_0 and f_0 .

综上，标度解极限行为的结果看来相当稳健。$\tilde{\rho} \to \infty$ 的极限行为仅依赖于无质量粒子。使用不同版本的流方程或不同形式的红外截断函数导致常数 $c_U(\tilde{\rho} \to \infty)$ 发生变化，这可以通过对 k 重标度，或是平移爱因斯坦框架中 φ 的定义来吸收。$u(\tilde{\rho} \to 0)$ 和 $f(\tilde{\rho} \to 0)$ 取恒定值这一结论似乎十分普适。u_0 和 f_0 的精确值通常依赖于紫外极限下未知的粒子内容，以及流方程和截断函数的具体实现方式。可以预期，一大类模型都会得到正的 u_0 和 f_0 。

Crossover Cosmology

跨界宇宙学

In view of the rather robust result for the limiting behavior of the flow equations, as well as for the associated cosmological field equations and their solutions, we investigate in this section in more detail the cosmology describing a crossover between the UV and IR fixed points. We focus on the scaling solution according to fundamental scale invariance and discuss possible modifications due to relevant parameters in the next section.

鉴于流方程的极限行为以及相关宇宙场方程与其解都得到了相当可靠的结果，我们在本节中更详细地研究描述紫外不动点与红外不动点之间跨界的宇宙学。我们聚焦于基于基本标度不变性的标度解，并在下一节讨论相关参数带来的可能修正。

Crossover Trajectory in Scale and Time

尺度与时间中的跨越轨迹

The scaling solution of quantum gravity exhibits both an ultraviolet (UV) and an infrared (IR) fixed point with the associated quantum scale symmetry. These fixed points are reached for $\tilde{\rho} \to 0$ and $\tilde{\rho} \to \infty$, respectively. For the scaling solution the functions $u(\tilde{\rho}), f(\tilde{\rho})$, and $K(\tilde{\rho})$ interpolate between these limits. Such solutions are called "crossover trajectories." They link two fixed points, describing a crossover from one fixed point behavior to a different one. Along a crossover trajectory the physics can change qualitatively. Close to the fixed points quantum scale symmetry remains a valid approximate symmetry. In regions of a qualitative variation with $\tilde{\rho}$, quantum scale symmetry no longer holds since the scale k appears indirectly through $\tilde{\rho}$.

量子引力的标度解同时具有紫外 (UV) 不动点和红外 (IR) 不动点，二者伴随相应的量子标度对称性。分别在 $\tilde{\rho} \to 0$ 和 $\tilde{\rho} \to \infty$ 处达到这两个不动点。对于该标度解，函数 $u(\tilde{\rho}), f(\tilde{\rho})$ 和 $K(\tilde{\rho})$ 在这两个极限之间插值。这类解被称为“跨越轨迹”。它们连接两个不动点，描述从一种不动点行为到另一种不同行为的跨越。沿跨越轨迹，物理性质可发生定性变化。在靠近不动点的区域，量子标度对称性仍是有效的近似对称性。在随 $\tilde{\rho}$ 发生定性变化的区域，量子标度对称性不再成立，因为尺度 k 通过 $\tilde{\rho}$ 间接显现。

One obvious crossover region corresponds to the qualitative change of $f(\tilde{\rho})$ from a constant to a linear increase with $\tilde{\rho}$. In the approximation

一个明显的跨越区域对应 $f(\tilde{\rho})$ 从恒定到随 $\tilde{\rho}$ 线性增长的定性变化。在近似下

$$f(\tilde{\rho}) = f_0 + 2\xi\tilde{\rho}, \tag{116}$$

the range of this crossover is given by $\tilde{\rho} \approx \tilde{\rho}_f$, with

该跨越的范围由 $\tilde{\rho} \approx \tilde{\rho}_f$ 给出，其中

$$\tilde{\rho}_f = \frac{f_0}{2\xi}. \tag{117}$$

The crossover in f describes the onset of the decoupling of the metric fluctuations. For $\tilde{\rho} \ll \tilde{\rho}_f$ the metric fluctuations play an important role for the flow equations. On the other hand, for $\tilde{\rho} \gg \tilde{\rho}_f$ the metric fluctuations decouple, their contribution being suppressed by powers of the small quantity $(\xi\tilde{\rho})^{-1}$. (This holds with the exception of a contribution to the $\tilde{\rho}$ -independent part of the flow of u .)

f 的跨越描述了度规涨落退耦的开始。对于 $\tilde{\rho} \ll \tilde{\rho}_f$ ，度规涨对流方程发挥重要作用。另一方面，对于 $\tilde{\rho} \gg \tilde{\rho}_f$ ，度规涨落发生退耦，其贡献被小量 $(\xi\tilde{\rho})^{-1}$ 的幂次压低。(这一结论存在例外: u 流中与 $\tilde{\rho}$ 无关的部分存在一项贡献。)

Another possible crossover concerns the behavior of the kinetial $K(\tilde{\rho})$. For the simplified ansatz

另一种可能的跨越涉及动力学项 $K(\tilde{\rho})$ 的行为。对于简化近似

$$K(\tilde{\rho}) = \frac{\kappa}{2\tilde{\rho}} + K_0, \tag{118}$$

the qualitative change occurs for

定性变化发生在

$$\tilde{\rho}_K = \frac{\kappa}{2K_0}. \tag{119}$$

With Eqs. (116) and (118) one has

结合式 (116) 和 (118)，可得

$$\begin{aligned} Z = \frac{\chi^2 K_E}{16M^2} &= \frac{K\tilde{\rho}}{8f} + \frac{3}{8f^2}\left(\tilde{\rho}\partial_{\tilde{\rho}} f\right)^2 \\ &= \frac{1}{16}\left[\frac{\kappa + 2K_0\tilde{\rho}}{f_0 + 2\xi\tilde{\rho}} + 6\left(1 + \frac{f_0}{2\xi\tilde{\rho}}\right)^{-2}\right], \end{aligned} \tag{120}$$

which interpolates - possibly in two steps - between the limits (45) and (27). We will see below how a crossover from $Z \gg 1$ to $Z \ll 1$ could be related to the end of the inflationary epoch in cosmology. Unfortunately, the parameters κ, K_0 , and ξ are not yet fixed by the present computations, nor is the approximate form (118) established.

它在极限 (45) 和 (27) 之间——可能分两步——插值。我们下文会看到，从 $Z \gg 1$ 到 $Z \ll 1$ 的跨越如何与宇宙学中暴胀时期的结束相关联。遗憾的是，参数 κ, K_0 和 ξ 尚未通过当前计算确定，近似形式 (118) 也未得到确立。

For the scaling solution of quantum gravity, the flow of coupling functions with k translates directly to the dependence of the effective action on the scalar field χ . In turn, the solutions of the field equations derived by variation of this effective action can translate the field dependence into a time dependence. We find typical

对于量子引力的标度解，耦合函数随 k 的流动直接转化为有效作用量对标量场 χ 的依赖。反过来，对该有效作用量变分得到的场方程解可将场依赖转化为时间依赖。我们发现典型的

"crossover cosmologies" for which the infinite past is characterized by $\chi \to 0$ and therefore to an approach to the UV fixed point, while the infinite future realizes $\chi \to \infty$ and therefore approaches the IR fixed point. The overall simple picture of cosmology is a crossover from the UV fixed point in the infinite past to the IR fixed point in the infinite future. The crossover cosmology typically happens in several stages that we will identify with inflation, kination, radiation domination, matter domination, and dark energy domination. For the detailed description of the matter and radiation domination epochs, particle physics is needed, in our context in the form of the scale-invariant standard model and extensions thereof. The inflation and kination epochs can be described by quantum gravity with a scalar field. (Extensions to several scalar fields are possible but will not be discussed in this note.)

“跨越宇宙学”，其无限过去由 $\chi \to 0$ 表征，因此趋近紫外不动点，而无限未来实现 $\chi \to \infty$，因此趋近红外不动点。宇宙学的整体简单图景就是从无限过去的紫外不动点到无限未来的红外不动点的跨越。跨越宇宙学通常分为多个阶段，我们可将其对应为暴胀、动力学期、辐射主导、物质主导和暗能量主导。要详细描述物质主导和辐射主导时期，需要粒子物理，在本文背景下以标度不变标准模型及其扩展形式呈现。暴胀和动力学时期可以通过带标量场的量子引力描述。(扩展到多个标量场是可行的，但本文不做讨论。)

Inflation and the Beginning Universe

暴胀与宇宙起源

A rather natural possible beginning of the universe is the close vicinity of the ultraviolet fixed point. Similarly to the infinite increase of k necessary to reach the fixed point precisely, an infinite increase of $-\eta$, which corresponds to an appropriate physical time [106], is needed for the cosmological solution to reach the fixed point. The universe is then eternal, with the fixed point realized precisely only in the infinite past. We discuss here in detail how the vicinity of the UV fixed point is related to inflation and how crossover away from the fixed point ends the inflationary epoch.

宇宙一个相当自然的可能开端，就是紫外固定点的临近区域。和精确抵达固定点需要 k 无限增长类似，要让宇宙学解抵达固定点，也需要对应物理时间 [106] 的 $-\eta$ 无限增长。因此宇宙是永恒的，只有在无限过去才会精确抵达固定点。我们在此详细讨论紫外固定点的临近区域和暴胀的关联，以及偏离固定点的跨接如何结束暴胀阶段。

Beginning at the Fixed Point

不动点处的开端

The UV fixed point $\chi = 0$ is an exact solution of the field equations. This follows generally from the discrete symmetry $\chi \to -\chi$, which only allows even powers of χ for all terms in the effective action. In the scaling frame for the metric, the cosmological evolution equations for $\chi = 0$ imply that geometry is given by de Sitter space with the conformal Hubble parameter

紫外不动点 $\chi = 0$ 是场方程的精确解。这一般可由离散对称性 $\chi \to -\chi$ 推导得出，该对称性仅允许有效作用量中所有项包含 χ 的偶次幂。在度规的标度框架下，$\chi = 0$ 的宇宙演化方程表明，几何由德西特空间给出，其共形哈勃参数为

$$\mathcal{H} = -\frac{1}{\eta},\ \chi = 0. \tag{121}$$

For cosmic time the Hubble parameter H is proportional to k,

就宇宙时间而言，哈勃参数 H 与 k 成正比,

$$H^2 = \frac{U}{3F} = \frac{u_0 k^2}{3 f_0}. \tag{122}$$

Equations (121), (122) are an exact solution of the field equations for our truncated effective action, provided $\widehat{V}(\chi = 0) > 0$. For a more general truncation the expression for H/k may be different, but the partial solution $\chi = 0$ remains. Also $\mathcal{H} = -1/\eta$, which reflects the scale symmetry of de Sitter space, is often realized.

若满足 $\widehat{V}(\chi = 0) > 0$ ，式 (121)、(122) 就是我们截断有效作用量下场方程的精确解。对于更一般的截断，H/k 的表达式可能不同，但偏解 $\chi = 0$ 仍然成立。此外，反映德西特空间标度对称性的 $\mathcal{H} = -1/\eta$ 通常也成立。

We will see that the solution (121), (122) is unstable with respect to small deviations. Arbitrarily small values of χ will grow as time increases. We describe the beginning stage of the universe by the close vicinity of the solution (121), (122). In the beginning stage only the space averaged field expectation values $g_{\mu\nu}$ and χ and the fluctuations of these fields encoded in correlation functions matter. Their evolution is very slow if measured in a "physical time" proportional to η . One may call this stage of the universe "Great Emptiness" [147]. In the infinite past $\eta \to -\infty$, the fixed point solution (122) is approached closer and closer. For $\eta \to -\infty$ all field expectation values vanish since the cosmic scale factor $a(\eta) = (-H\eta)^{-1}$ goes to zero and therefore $g_{\mu\nu} \to 0$. Only the correlation functions differ from zero in this "symmetric" or "ageometric" state.

我们会看到，解 (121)、(122) 对小扰动是不稳定的，任意微小的 χ 值都会随时间推移增长。我们将宇宙的初始阶段描述为非常接近解 (121)、(122) 的状态。在初始阶段，只有空间平均场期望值 $g_{\mu\nu}$ 和 χ ，以及关联函数所编码的这些场的涨落发挥作用。如果用与 η 成正比的“物理时间”度量，它们的演化非常缓慢。这个宇宙阶段可以被称为“大虚空”[147]。在无限过去 $\eta \to -\infty$ ，不动点解 (122) 会被越来越逼近。在 $\eta \to -\infty$ 处，所有场期望值都为零，因为宇宙尺度因子 $a(\eta) = (-H\eta)^{-1}$ 趋近于零，因此 $g_{\mu\nu} \to 0$ 。在这个“对称态”或“无几何态”中，只有关联函数非零。

Inflationary Cosmology

暴胀宇宙学

The vicinity of the solution (121), (122) corresponds to an epoch of inflationary cosmology, as we will discuss in more detail here. This simple beginning requires $u_0 > 0$ and $f_0 > 0$. These conditions are not realized for the standard model of particle physics coupled to quantum gravity for which one finds $u_0 < 0, f_0 > 0$. We will assume here that the standard model is extended to some grand unified theory at some unification scale much larger than the Fermi scale. Due to the large number of bosonic fields, one finds positive u_0 for SO (10) - or SU (5) -unification. Any other extension leading to positive u_0 is possible as well.

解 (121)、(122) 的邻域对应暴胀宇宙学的一个演化时期，我们将在此展开详细讨论。这个简单的初始条件要求 $u_0 > 0$ 和 $f_0 > 0$ 。对于耦合量子引力的粒子物理标准模型，这些条件无法满足，该模型给出 $u_0 < 0, f_0 > 0$ 。我们在此假设，标准模型在远高于费米能标的统一能标下被扩展为某种大统一理论。由于存在大量玻色场，对于 SO (10) 统一或 SU (5) 统一，可以得到正的 u_0 。任何其他能得到正 u_0 的扩展模型也都是可行的。

We will discuss inflation in the Einstein frame, since this is most familiar. Many simple features, as the presence of a UV fixed point and the associated quantum scale symmetry, are no longer directly visible in the Einstein frame. Also the field transformation of the Weyl scaling introduces a mass scale M which is not an intrinsic mass scale for the scaling solution in quantum gravity. On the other hand, the field equations in the Einstein frame take a simple form where one does not need to take the variation of masses into account.

我们将在爱因斯坦框架下讨论暴胀，因为这是大家最熟悉的表述。许多关键性质，比如紫外不动点的存在以及相关的量子标度对称性，在爱因斯坦框架下无法再直接显现。此外，外尔标度的场变换会引入质量标度 M ，该质量标度并非量子引力中标度解的内禀质量标度。另一方面，爱因斯坦框架下的场方程形式简洁，无需考虑质量的变分。

The homogeneous field equations in the Einstein frame take the form

爱因斯坦框架下的均匀场方程形式为

$$H^2 = \frac{1}{3M^2}\left(U_E + \frac{Z}{2}(\partial_t\varphi)^2\right)$$

$$\left(\partial_t^2 + 3H\partial_t\right)\varphi + \frac{\eta_Z}{8M}(\partial_t\varphi)^2 + \frac{1}{Z}\frac{\partial U_E}{\partial\varphi} = 0, \tag{123}$$

where

其中

$$\eta_Z = 4M\frac{\partial \ln Z}{\partial\varphi} = \frac{\partial \ln Z}{\partial \ln \chi}. \tag{124}$$

The Z -factor can be absorbed by using the "canonical scalar field" σ with the canonical kinetic term, defined by

Z 因子可以通过带有正则动能项的“正则标量场” σ 吸收，其定义为

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\varphi} = Z^{1/2}(\varphi) \tag{125}$$

For the canonical field the equivalent field equations become

对于正则场，等价场方程变为

$$H^2 = \frac{1}{3M^2}\left(U_E + \frac{1}{2}(\partial_t\sigma)^2\right)$$

$$\left(\partial_t^2 + 3H\partial_t\right)\sigma = -\frac{\partial U_E}{\partial\sigma} \tag{126}$$

On the other hand, for a standard form of the potential, the physics is encoded in $Z(\varphi)$ which often allows for a simple description [52,133].

另一方面，对于标准形式的势，物理信息被编码在 $Z(\varphi)$ 中，这通常可以简化描述 [52,133]。

The inflationary epoch is characterized by a slow evolution of the scalar field ("slow roll") such that the term $\sim (\partial_t\sigma)^2$ in Eq. (126) is small as compared to the almost constant U_E . Then H is approximately constant such that the expansion becomes exponential or some other very fast increase. For the fixed point solution (122), the slow-roll approximation $\dot{H}/H^2 \ll 1, (\partial_t\sigma)^2 \ll U_E$ becomes exact. In the Einstein frame this corresponds to $\varphi \to -\infty$ for which the potential approaches a constant value

暴胀时期的特征是标量场演化缓慢（“慢滚”），因此式 (126) 中的 $\sim (\partial_t\sigma)^2$ 项远小于近似为常数的 U_E 。此时 H 近似为常数，膨胀呈指数形式或其他极快速增长形式。对于不动点解 (122)，慢滚近似 $\dot{H}/H^2 \ll 1, (\partial_t\sigma)^2 \ll U_E$ 是精确成立的。在爱因斯坦框架下，这对应势趋近于常数的 $\varphi \to -\infty$

$$U_E \to \frac{u_0 M^4}{f_0^2} \tag{127}$$

In the vicinity of the UV fixed point the slow-roll approximation remains valid.

在紫外不动点的邻域内，慢滚近似仍然成立。

The slow-roll approximation is characterized by two small parameters

慢滚近似由两个小参数刻画

$$\varepsilon = \frac{M^2}{2}\left(\frac{\partial \ln U_E}{\partial\sigma}\right)^2$$

$$\eta = \frac{M^2}{U_E}\frac{\partial^2 U_E}{\partial\sigma^2} = 2\varepsilon + M^2\frac{\partial^2 \ln U_E}{\partial\sigma^2}. \tag{128}$$

Inflation ends once ε or η reach values of the order one. In terms of φ or $\tilde{\rho}$ the slow-roll parameters are given by

当 ε 或 η 达到量级为 1 的值时，暴胀结束。用 φ 或 $\tilde{\rho}$ 表示的慢滚参数为

$$\varepsilon = \frac{1}{2Z}\left(M\frac{\partial \ln U_E}{\partial\varphi}\right)^2 = \frac{1}{8Z}\left[\tilde{\rho}\partial_{\tilde{\rho}}\ln\left(\frac{u}{f^2}\right)\right]^2,$$

$$= \frac{1}{2Z}\left(\frac{c_F}{f} - \frac{c_U}{u}\right)^2, \tag{129}$$

and

且

$$\eta = 2\varepsilon + \frac{M^2}{Z}\frac{\partial^2 \ln U_E}{\partial\varphi^2} - \frac{\eta_Z}{8Z}M\frac{\partial \ln U_E}{\partial\varphi}$$

$$= 2\varepsilon + \frac{1}{4Z}\left[\tilde{\rho}^2\partial^2_{\tilde{\rho}} + \left(1 - \frac{\eta_Z}{4}\right)\tilde{\rho}\partial_{\tilde{\rho}}\right]\ln\left(\frac{u}{f^2}\right)$$

$$= \frac{1}{Z}\left\{\left(1 + \frac{\eta_Z}{8}\right)\frac{c_U}{u} - \frac{1}{2}\left(1 + \frac{\eta_Z}{4}\right)\frac{c_F}{f}\right.$$

$$+\frac{3c_F^2}{2f^2}-\frac{2c_Fc_U}{fu}+\frac{1}{2f}\tilde{\rho}\partial_{\tilde{\rho}}c_F-\frac{1}{2u}\tilde{\rho}\partial_{\tilde{\rho}}c_U\bigg\}. \tag{130}$$

For the last relations for ε and η , we have employed the scaling equation (81). This demonstrates directly that the slow-roll parameters are calculable in terms of the scaling solution of the flow equations!

对于最后得到的 ε 和 η 的关系，我们利用了标度方程 (81)。这直接表明，慢滚参数可以通过流方程的标度解计算得到!

For $\varphi\to-\infty$ or $\tilde{\rho}\to 0$ we may use the limit of the scaling solution

对于 $\varphi\to-\infty$ 或 $\tilde{\rho}\to 0$ ，我们可以使用标度解的极限

$$u=u_0+\widetilde{m}_0^2\tilde{\rho},\ f=f_0+2\xi_0\tilde{\rho}, \tag{131}$$

or

$$U_E=\frac{M^4u}{f^2}=\frac{M^4u_0}{f_0^2}\left[1+\beta_0\tilde{\rho}\right],$$

$$\beta_0=\frac{\widetilde{m}_0^2}{u_0}-\frac{4\xi_0}{f_0^2} \tag{132}$$

With

结合

$$\tilde{\rho}=\frac{\chi^2}{2k^2}=\frac{1}{2}\exp\left(\frac{\varphi}{2M}\right),\ M\frac{\partial\ln U_E}{\partial\varphi}=\frac{\tilde{\rho}}{2}\frac{\partial\ln U_E}{\partial\tilde{\rho}}, \tag{133}$$

one finds for $\tilde{\rho}\to 0$ or $\varphi\to-\infty$

针对 $\tilde{\rho}\to 0$ 或 $\varphi\to-\infty$ ，可得

$$M\frac{\partial\ln U_E}{\partial\varphi}=\frac{\beta_0\tilde{\rho}}{2}=\frac{\beta_0}{4}\exp\left(\frac{\varphi}{2M}\right). \tag{134}$$

Thus, ε and η vanish exponentially for $\varphi\to-\infty$. We conclude that the scaling solution predicts an inflationary epoch in cosmology.

因此，对于 $\varphi\to-\infty$ ， ε 和 η 指数趋近于零。我们得出结论: 该标度解预测宇宙学中存在一个暴胀时期。

End of Inflation by Kinetial Crossover

动力学交叉引发的暴胀结束

We have argued in section ”Quantum Scale Symmetry” that the observed small size of the primordial cosmic fluctuations requires that fluctuations decouple when φ is already larger than M , unless the ratio

u_0/f_0^2 is tiny. In this case the potential is already exponentially decreasing according to Eq. (26), providing for a natural suppression factor for the fluctuations. In the range of validity of Eq. (26), one has $M\partial \ln U_E/\partial\varphi = -1$, such that ε and η only depend on Z [138]:

我们已经在“量子标度对称性”一节中指出，观测到的原初宇宙涨落幅度很小，这要求涨落在 φ 已经大于 M 时退耦合，除非比值 u_0/f_0^2 极小。在这种情况下，根据式 (26)，势场已经呈指数下降，为涨落提供了自然的压制因子。在式 (26) 的有效范围内，可得 $M\partial \ln U_E/\partial\varphi = -1$ ，因此 ε 和 η 仅依赖于 Z [138]:

$$\varepsilon = \frac{1}{2Z},\ \eta = \frac{1+8\eta_Z}{Z}. \tag{135}$$

These simple relations make a discussion of inflationary cosmology in terms of the kinetial $Z(\varphi)$ very convenient.

这些简单关系使得用动力学 $Z(\varphi)$ 讨论暴胀宇宙学十分方便。

In the approximation (135) an inflationary epoch lasts as long as Z remains larger than one. If the end of inflation occurs for a range of φ for which the exponential decrease (26) is valid, there is a possibility to associate the end of inflation with a crossover in the kinetial (120)

在近似式 (135) 中，只要 Z 保持大于 1，暴胀时期就会持续。如果暴胀结束发生在满足指数下降式 (26) 有效的 φ 范围内，就可以将暴胀的结束和动力学 (120) 中的突变联系起来

$$Z = \frac{3}{8} + \frac{K_0}{16\xi} + \frac{\kappa}{32\xi\tilde{\rho}}. \tag{136}$$

Values of Z smaller than one can happen for large enough $\tilde{\rho}$, in particular for negative K_0 or, more generally, if $K(\tilde{\rho})$ reaches negative values. It is conceivable that a crossover in Z happens for values of $\tilde{\rho}$ much larger than one. This could explain a small value

Z 小于 1 的情况可以在 $\tilde{\rho}$ 足够大时出现，尤其是当 K_0 为负时，或者更一般地，当 $K(\tilde{\rho})$ 取负值时。可以想见，Z 会在 $\tilde{\rho}$ 远大于 1 时发生突变，这可以解释为何数值很小

$$\frac{U_E}{M^4} = \frac{u}{f^2} = \frac{u}{4\xi^2\tilde{\rho}^2} \tag{137}$$

and therefore a small amplitude of the primordial fluctuations (49).

因此原初涨落的幅度也很小 (49)。

End of Inflation by Grand Unified Threshold

大统一阈值引发的暴胀结束

The approximation (135) is valid if $u(\tilde{\rho})$ and $f(\tilde{\rho})/\tilde{\rho}$ are approximately constant. This may not hold in threshold regions where some of the particles decouple due to their $\tilde{\rho}$ -dependent mass. We will next argue that a threshold region is a good candidate for ending inflation. In view of the small observed fluctuation amplitude, we can use $u/f^2 \ll 1$ and $|c_F/f| \ll 1$ in order to simplify the general expressions (129), (130) for the slow-roll parameters

若 $u(\tilde{\rho})$ 和 $f(\tilde{\rho})/\tilde{\rho}$ 近似为常数，近似式 (135) 成立。在部分粒子因依赖 $\tilde{\rho}$ 的质量退耦的阈值区域，这一条件可能不成立。我们接下来将论证，阈值区域是结束暴胀的合理候选。考虑到观测到的涨落振幅很小，我们可以利用 $u/f^2 \ll 1$ 和 $|c_F/f| \ll 1$ 来化简慢滚参数的通式 (129)、(130)

$$\varepsilon = \frac{c_U^2}{2Zu^2}$$

$$\eta = \frac{1}{Z}\left\{\left(1 + \frac{\eta_Z}{8}\right)\frac{c_U}{u} - \frac{1}{2u}\tilde{\rho}\partial_{\tilde{\rho}}c_U\right\}. \tag{138}$$

In the flat regions for u one has $u = c_U =$ const. and we recover Eq. (135). In the threshold regions c_U/u can differ from one substantially, however. For $|c_F/f| \ll 1$ one has

在 u 的平坦区域中，$u = c_U =$ 为常数，我们可以得到式 (135)。但在阈值区域，c_U/u 可以和 1 有明显差异。对 $|c_F/f| \ll 1$ 有

$$c_U = \frac{\bar{N}_U(\tilde{\rho})}{128\pi^2},\ \bar{N}_U = 2 + \bar{N}_S + 2\bar{N}_V - 2\bar{N}_F, \tag{139}$$

where the effective particle numbers can change rather rapidly with $\tilde{\rho}$.

其中有效粒子数会随 $\tilde{\rho}$ 发生相当快的变化。

As an example we consider the variation of c_U for the transition from some grand unified theory (GUT) to the standard model (SM) at a scale

我们以某大统一理论 (GUT) 到标准模型 (SM) 在某能标下的转变为例，分析 c_U 的变化

$$m_X = \delta\sqrt{F},\ \tilde{m}_X^2 = \frac{m_X^2}{k^2} = 2\delta^2\xi\tilde{\rho}. \tag{140}$$

In the Einstein frame the small parameter δ denotes the ratio of the GUT scale M_X over the Planck mass M . For the standard model + cosmon one has $\bar{N}_S = 5$, $\bar{N}_V = 12, \bar{N}_F = 45$, and therefore negative $\bar{N}_U$,

在爱因斯坦框架中，小参数 δ 表示 GUT 能标 M_X 与普朗克质量 M 的比值。对于标准模型 + 宇宙标量场，有 $\bar{N}_S = 5$ 、$\bar{N}_V = 12, \bar{N}_F = 45$ ，因此 $\bar{N}_U$ 为负，

$$\bar{N}_U^{(\mathrm{SM})} = 2 + 5 + 24 - 90 = -59, \tag{141}$$

while SO (10) -unification implies $\bar{N}_V = 45, \bar{N}_F = 48$, leading to positive $\bar{N}_U$,

而 SO (10) 统一意味着 $\bar{N}_V = 45, \bar{N}_F = 48$ ，因此 $\bar{N}_U$ 为正，

$$\bar{N}_U^{(\mathrm{GUT})} = \bar{N}_S^{(\mathrm{GUT})} - 6. \tag{142}$$

The number of scalars $\bar{N}_S^{(\mathrm{GUT})}$ depends on the particular SO(10)-model and is typically large.

标量场的数量 $\bar{N}_S^{(\mathrm{GUT})}$ 取决于具体的 SO(10) 模型，通常数值较大。

For a simplified model of the threshold, all non-SM particles are taken to have the same mass, resulting in

对于该阈值的简化模型，假设所有非标准模型粒子质量相同，可得

$$\bar{N}_U(\tilde{\rho}) = \frac{\bar{N}_S^{(\mathrm{GUT})} + 53}{1 + \widetilde{m}_X^2} - 59, \tag{143}$$

or

$$c_U = \frac{1}{128\pi^2}\left(\frac{A}{1+\tau\tilde{\rho}} + B\right), \tag{144}$$

with

其中

$$A = \bar{N}_S^{(\mathrm{GUT})} + 53,\ B = -59,\ \tau = 2\delta^2\xi. \tag{145}$$

Away from the threshold region where $\tau\tilde{\rho} \approx 1$, one expects flat regions, with $u = (A+B)/\left(128\pi^2\right)$ for $\tau\tilde{\rho} \ll 1$ and $u = B/\left(128\pi^2\right)$ for $\tau\tilde{\rho} \gg 1$. Since u changes from positive to negative values as $\tilde{\rho}$ increases, there will be a region where $u \approx 0$ and therefore $\varepsilon \gg 1$. Such a threshold will end the inflationary epoch.

在远离阈值区域处满足 $\tau\tilde{\rho} \approx 1$ ，可以预期存在平坦区域，对 $\tau\tilde{\rho} \ll 1$ 有 $u = (A+B)/\left(128\pi^2\right)$ ，对 $\tau\tilde{\rho} \gg 1$ 有 $u = B/\left(128\pi^2\right)$ 。随着 $\tilde{\rho}$ 增大，u 从正值变为负值，因此必然存在一个区域满足 $u \approx 0$ ，进而满足 $\varepsilon \gg 1$ 。这类阈值会终结暴胀时期。

Threshold Behavior

阈值行为

For a more detailed picture we need to compute the $\tilde{\rho}$ -dependence of u through the threshold region (93), (95),

为得到更细致的图像，我们需要计算阈值区域 (93)、(95) 中 $\tilde{\rho}$ 对 u 的依赖关系，

$$u = \frac{1}{128\pi^2}\left(At_u(\tau\tilde{\rho}) + B\right). \tag{146}$$

It interpolates smoothly between

它在以下区间之间平滑插值

$$u(\tau\tilde{\rho} \ll 1) = \frac{A+B}{128\pi^2},\ u(\tau\tilde{\rho} \gg 1) = \frac{B}{128\pi^2}. \tag{147}$$

The parameter τ sets only the position of the threshold, since it can be absorbed by a shift in $x = \ln\tilde{\rho}$. For $B < 0 < A + Bu$ changes sign at $\bar{x}$ where c_U is still positive. Indeed, with $c_U - u = -\frac{1}{2}\tilde{\rho}\partial_{\tilde{\rho}}u$ the r.h.s. is positive if $u(\tilde{\rho})$ is decreasing. Starting from small $\tilde{\rho}$ both u and c_U decrease while c_U/u increases for increasing $\tilde{\rho}$

参数 τ 仅决定阈值的位置，因为它可以被 $x = \ln\tilde{\rho}$ 的平移吸收。对于 $B < 0 < A + Bu$ ，它在 $\bar{x}$ 处变号，此时 c_U 仍为正。确实，当满足 $c_U - u = -\frac{1}{2}\tilde{\rho}\partial_{\tilde{\rho}}u$ 时，若 $u(\tilde{\rho})$ 递减，右侧为正。从小 $\tilde{\rho}$ 开始，随着 $\tilde{\rho}$ 增大，u 和 c_U 均递减，而 c_U/u 递增

$$\frac{c_U}{u} = 1 + \frac{A\tau\tilde{\rho}}{A+B} \tag{148}$$

The increase of c_U/u continues until it diverges as $u \to 0$ for $\tilde{\rho} \to \overline{\rho}$. The slow-roll approximation breaks down for $\tilde{\rho} < \overline{\rho}$. For $\tilde{\rho} > \overline{\rho}$ the potential u becomes negative and $\tilde{u}$ approaches the negative constant B for $\tilde{\rho} \to \infty$. The coefficient c_U changes signs for $\tilde{\rho} > \overline{\rho}$.

c_U/u 持续增大，直到在 $\tilde{\rho} \to \overline{\rho}$ 处随 $u \to 0$ 发散。慢滚近似在 $\tilde{\rho} < \overline{\rho}$ 处失效。对于 $\tilde{\rho} > \overline{\rho}$ ，势 u 变为负，且当 $\tilde{\rho} \to \infty$ 时 $\tilde{u}$ 趋近于负常数 B 。系数 c_U 在 $\tilde{\rho} > \overline{\rho}$ 处变号。

As a consequence, the frame-invariant potential $\hat{V}$ or the potential in the Einstein frame $U_E = M^4\hat{V}$ exhibits a shallow minimum for $\tilde{\rho} > \overline{\rho}$. We show this in Fig. 2 for a typical grand unified theory based on SO(10). Realistic cosmology has to avoid that the scalar field settles in this minimum after inflation. Otherwise, a substantial negative cosmological constant would stop further expansion.

因此，规范不变势 $\hat{V}$ 或爱因斯坦框架下的势 $U_E = M^4\hat{V}$ 在 $\tilde{\rho} > \overline{\rho}$ 区域存在一个浅极小值。我们在图 2 中展示了基于 SO(10) 的典型大统一理论的这一特征。现实宇宙学必须避免暴胀后标量场停留在这个极小值中，否则显著的负宇宙学常数会终止进一步的膨胀。

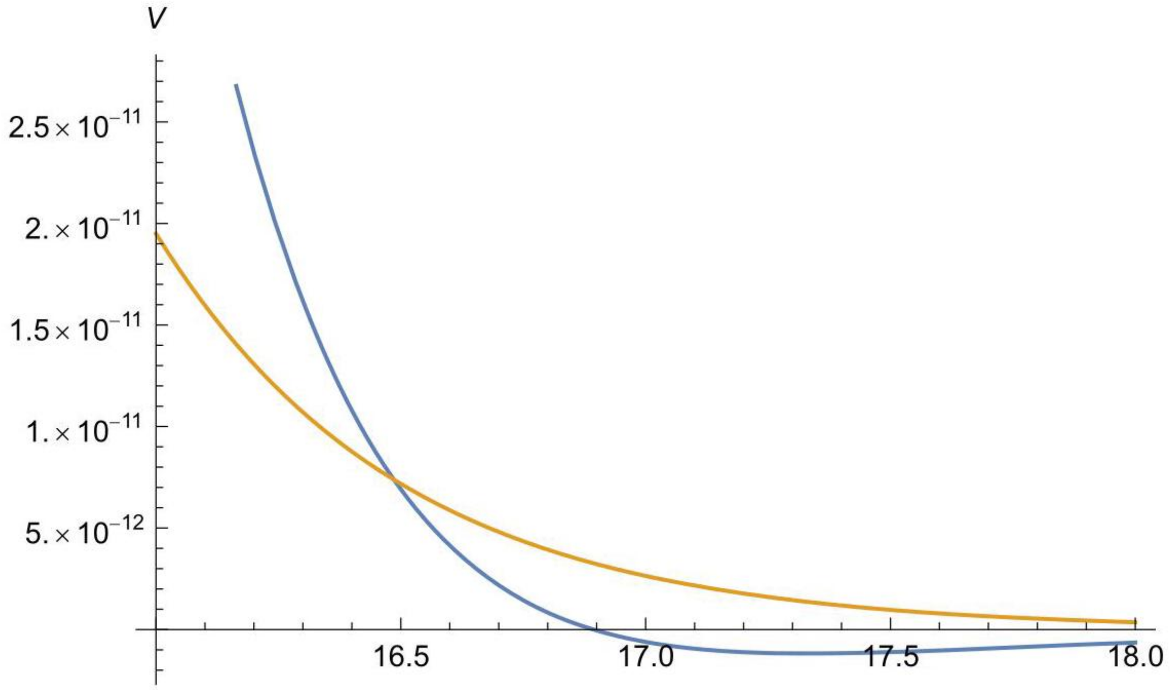

Fig. 2 Minimum of scalar potential for grand unified theory. We show $\hat{V}$ as a function of $x = \ln\tilde{\rho} = \varphi/(2M)$ in a range of large x. One observes the shallow minimum for the blue curve. For comparison, we also plot the potential that would be obtained by moving the Fermi scale to the grand unified scale (orange curve, for $\hat{V}/10$ for better visibility)

图 2 大统一理论中标量势的极小值。我们展示了大 x 范围内 $\hat{V}$ 作为 $x = \ln\tilde{\rho} = \varphi/(2M)$ 的函数。可以看到蓝色曲线存在浅极小值。为便于比较，我们也绘制了将费米标度移动到大统一标度得到的势(橙色曲线，为提升可见性已对 $\hat{V}/10$ 做处理)

Grand Unified Scale

大统一标度

The scaling solution predicts that inflation ends at the latest during the GUT-phase transition when u turns negative due to the large number of imbalance between fermions and bosons in the standard model. This relates naturally the small fluctuation amplitude to the small ratio δ between the unification mass M_X and the Planck mass M . Indeed, the characteristic range for this threshold and therefore the end of inflation is given by $\tilde{\rho}_e \approx \tau^{-1} = \left(2\delta^2\xi\right)^{-1}$. At this value of $\tilde{\rho}_e$ one has

标度解预测，暴胀最晚在大统一理论相变期间结束，此时因标准模型中费米子与玻色子的数量差很大，u 变为负值。这自然将小涨落振幅与统一质量 M_X 和普朗克质量 M 之间的小比值 δ 联系起来。事实上，该阈值 (即暴胀的终点) 的特征范围由 $\tilde{\rho}_e \approx \tau^{-1} = \left(2\delta^2\xi\right)^{-1}$ 给出。当 $\tilde{\rho}_e$ 取该值时，有

$$\hat{V}_e = \frac{u_e}{4\xi^2\tilde{\rho}_e^2} \approx \delta^4 u_e, \tag{149}$$

with $u_e = u(\tilde{\rho}_e)$ and ξ characterizing $f(\tilde{\rho}_e) = 2\xi\tilde{\rho}_e$. Typical values $M_X \approx 10^{16}\mathrm{GeV}$, with $\delta \approx 10^{-2}$, are well compatible with grand unified models and lead to a realistic amplitude of the primordial cosmic fluctuations.

其中 $u_e = u(\tilde{\rho}_e)$ 与 ξ 表征 $f(\tilde{\rho}_e) = 2\xi\tilde{\rho}_e$ 。在 $\delta \approx 10^{-2}$ 的条件下，典型值 $M_X \approx 10^{16}\mathrm{GeV}$ 与大统一模型完全相容，且能得到符合实际的原初宇宙涨落振幅。

Primordial Fluctuation Spectrum

原初涨落谱

The quantum effective action determines the full propagator or the connected two-point correlation function as the inverse of its second functional derivative. For a Minkowski signature taking the inverse amounts to an initial value problem [137]. One may assume that the propagator takes for very large (comoving and covariant) momenta the same Lorentz-covariant form as for flat space. This is well motivated since effects of a non-flat geometry become suppressed for wavelengths of the fluctuations much smaller than characteristic geometric length scales. With this assumption the propagators for the physical metric fluctuations and scalar fluctuations are the same as usually obtained by canonical quantization in a Bunch-Davies vacuum [109]. The amplitude and spectrum of the primordial cosmic fluctuations are directly given by the full propagator. Computing the quantum effective action therefore gives direct quantitative access to the cosmic fluctuation spectrum [137,139,141]. We will assume here the approximation (3) with F, U , and K according to the scaling solution for quantum gravity.

量子有效作用量可确定完整传播子，也就是连通两点关联函数，对应其二阶泛函导数的逆。对于闵氏号差，求逆等价于一个初值问题 [137]。我们可以假设，对于极大的 (共动且协变) 动量，传播子具有和平直空间相同的洛伦兹协变形式。该假设具有充分合理性: 当涨落的波长远小于特征几何长度尺度时，非平坦几何的效应会受到抑制。在此假设下，物理度量涨落和标量涨落的传播子与通常在邦奇-戴维斯真空中通过正则量子化得到的结果一致 [109]。原初宇宙涨落的振幅和谱由完整传播子直接给出。因此，计算量子有效作用量即可直接定量得到宇宙涨落谱 [137,139,141]。我们在此使用近似式 (3)，其中 F, U 和 K 遵循量子引力标度解。

The cosmic fluctuation spectrum and the amplitude of the fluctuations are independent of the metric frame [139]. We can therefore turn immediately to the well-known results for the inflationary epoch in the Einstein frame. The amplitude of the scalar fluctuations $\mathcal{A}$ and tensor fluctuations $r\mathcal{A}$ is given by Eqs. (47)-(49), where $\mathcal{A}$ can be compared with observation. For inflationary cosmology the tensor to scalar ratio r is given by

宇宙涨落谱和涨落振幅与度量框架无关 [139]。因此我们可以直接使用爱因斯坦框架下 inflation 时期的已有成熟结果。标量涨落振幅 $\mathcal{A}$ 和张量涨落振幅 $r\mathcal{A}$ 由式 (47)-(49) 给出，其中 $\mathcal{A}$ 可与观测结果对比。对于暴涨宇宙学，张标比 r 可表示为

$$r = 16\varepsilon \,. \tag{150}$$

Furthermore, the momentum dependence of the scalar fluctuations is determined by a spectral index

此外，标量涨落的动量依赖由谱指数决定

$$n = 1 - 6\varepsilon + 2\eta. \tag{151}$$

Once $Z(\varphi)$ is computed for the scaling solution, both r and n are determined for a given particle content. Observation of the primordial fluctuation spectrum can be used directly for a test of models.

一旦得到标度解的 $Z(\varphi)$，给定粒子含量下 r 和 n 即可完全确定。原初涨落谱的观测结果可直接用于模型检验。

The quantities $U_E/M^4, \varepsilon$, and η relevant for the observed fluctuations of the CMB have to be evaluated at a time when their wavelengths decouple at a certain number N of e -foldings before the end of inflation (typically $N \approx 50-70$ depending on the heating after inflation). In the Einstein frame this decoupling happens when the corresponding wavelength exceeds the horizon. The same decoupling happens in all metric frames even though no geometric horizon may be present. Computing $\varphi(N)$ at this time expresses r and n as functions of N. For this purpose one replaces the time variable t by the number of e -foldings, with a_f the scale factor at the end of inflation

与 CMB 观测涨落相关的物理量 $U_E/M^4, \varepsilon$ 和 η，需要在涨落波长退耦时计算，对应 inflation 结束前的 e 折叠数为 N (通常为 $N \approx 50-70$，具体取决于 inflation 后的加热过程)。在爱因斯坦框架中，当对应波长超过视界时发生退耦。即使不存在几何视界，所有度量框架中都会发生相同的退耦过程。计算此时的 $\varphi(N)$ 即可得到作为 N 函数的 r 和 n。为此我们将时间变量 t 替换为 e 折叠数，其中 a_f 是 inflation 结束时的标度因子

$$N = -\ln\left(\frac{a}{a_f}\right), \quad \frac{\mathrm{d}N}{\mathrm{d}t} = -H. \tag{152}$$

The field equation in the slow-roll approximation reads

慢滚近似下的场方程为

$$\frac{\partial}{\partial N}\left(\frac{\sigma}{M}\right) = M\frac{\partial \ln U_E}{\partial \sigma}, \tag{153}$$

and one infers the expression (with φ_f the value of φ at the end of inflation)

由此可推得下述表达式 (其中 φ_f 是 inflation 结束时 φ 的值)

$$\begin{aligned} N &= \frac{1}{M}\int_{\varphi}^{\varphi_f} \mathrm{d}\varphi' Z(\varphi')\left(-M\frac{\partial \ln U_E}{\partial \varphi'}\right)^{-1} \\ &\approx \frac{1}{M}\int_{\varphi}^{\varphi_f} \mathrm{d}\varphi' Z(\varphi')\,\frac{u}{c_U}(\varphi'). \end{aligned} \tag{154}$$

This expression simplifies [138] for flat regions in u where $u = c_U$.

对于满足 $u = c_U$ 的 u 平坦区域，该表达式可简化 [138]

In the approximation (135) one has $\varepsilon = 8/Z$ and $n = 1 - (1 - 16\eta_Z)/Z$. This requires a large value of Z at the time of decoupling of the fluctuations. Realistic values of $r/(1-n)$ also demand substantial negative η_Z . The relations (135) will be modified according to Eq. (138), if decoupling occurs in a threshold region. Given the high predictivity of the scaling solution for quantum gravity, it is possible that the simplest model of the metric coupled to a single scalar field may be falsified by observation once a computation of the kinetial becomes available. One may then have to proceed to extensions with more than one scalar field or invariants with more than two derivatives playing a role during inflation.

在近似式 (135) 中可得 $\varepsilon = 8/Z$ 和 $n = 1 - (1 - 16\eta_Z)/Z$ 。这要求涨落退耦时 Z 取较大值。realistic 的 $r/(1-n)$ 值也要求 η_Z 显著为负。若退耦发生在阈值区域，关系式 (135) 需要根据式 (138) 修正。量子引力的标度解具有很高的预测性，一旦完成动力学项的计算，度量耦合单个标量场的最简模型就有可能被观测证伪。这种情况下我们就需要扩展模型，引入多个标量场，或是考虑 inflation 时期发挥作用的二阶以上导数不变量。

Kination

动能主导期

The inflationary epoch is typically followed by an epoch for which the scalar kinetic energy dominates, while radiation and matter are still negligible.

膨胀时期之后通常会进入一个标量动能占主导的阶段，此时辐射和物质的占比仍然可以忽略。

Scaling Solution for Kination

动力学阶段标度解

The kination epoch can be characterized by a scaling solution [128]. For this solution the field σ changes logarithmically with time, such that the kinetic energy decreases $\sim t^{-1}$. We make the ansatz

动力学阶段可以用标度解表征 [128]。对该解，场 σ 随时间对数变化，因此动能密度呈 $\sim t^{-1}$ 减小。我们做如下假设

$$\sigma = cM \ln\left(\frac{t}{t_{\text{kin}}}\right), \; H = \eta_H t^{-1}. \tag{155}$$

For this ansatz the field equations take the form

根据该假设，场方程可写为

$$\eta_H^2 = \frac{c^2}{6} + \frac{U_E t^2}{3M^2}$$

$$3\eta_H - 1 = -\frac{t^2}{cM}\frac{\partial U_E}{\partial \sigma}. \tag{156}$$

As long as the potential U_E can be neglected, the solution for the kination period reads

只要势场 U_E 可以忽略，动力学阶段的解就为

$$\eta_H = \frac{1}{3},\ c^2 = \frac{2}{3}. \tag{157}$$

The kination approximation (155)-(157) remains valid as long as the contribution from the effective potential or from radiation and nonrelativistic matter remains negligible. In the limit of constant Z , one has $\varphi = Z^{-1/2}\sigma$ and therefore

只要有效势的贡献，或是辐射与非相对论物质的贡献仍可忽略，近似式 (155)-(157) 就成立。在 Z 为常数的极限下，可得 $\varphi = Z^{-1/2}\sigma$ ，因此

$$U_E \sim M^4 \exp\left(-\frac{\varphi}{M}\right)$$

$$= M^4 \exp\left(-\frac{\sigma}{\sqrt{Z}M}\right) \sim \left(\frac{t}{t_{\text{kin}}}\right)^{-\sqrt{\frac{2}{3Z}}}. \tag{158}$$

Thus, U_E decays faster than t^{-2} if $Z < 1/6$. If such a regime with constant $Z < 1/6$ is reached the potential becomes less and less important as t increases. This also holds for

因此，若满足 $Z < 1/6$ ，则 U_E 衰减快于 t^{-2} 。如果进入了 $Z < 1/6$ 为常数的区域，随着 t 增大，势场的重要性会越来越低。这也适用于

$$M\frac{\partial U_E}{\partial \sigma} = -\frac{U_E}{\sqrt{Z}}. \tag{159}$$

On the other hand, in the approximation of constant $\eta_Z = 4M\partial \ln Z/\partial\varphi$, one has

另一方面，在 $\eta_Z = 4M\partial \ln Z/\partial\varphi$ 为常数的近似下，可得

$$Z = \bar{Z}\exp\left(\frac{\eta_Z\varphi}{4M}\right)$$

$$\sigma = \frac{8M\bar{Z}^{1/2}}{\eta_Z}\exp\left(\frac{\eta_Z\varphi}{8M}\right) + \sigma_0 \tag{160}$$

For $\eta_Z < 0$ the integration constant σ_0 is the value of σ that is reached for $\varphi \to \infty$. With

对 $\eta_Z < 0$ ，积分常数 σ_0 就是 $\varphi \to \infty$ 处对应的 σ 取值。结合

$$U_E \sim \exp\left(-\frac{\varphi}{M}\right) = \left[\frac{\eta_Z}{8M\sqrt{\bar{Z}}}(\sigma - \sigma_0)\right]^{-\frac{8}{\eta_Z}}, \tag{161}$$

$$M\frac{\partial U_E}{\partial \sigma} = -\frac{U_E}{\sqrt{Z}} \sim \bar{Z}^{-\frac{1}{2}}\left[\frac{\eta_Z}{8M\sqrt{\bar{Z}}}(\sigma - \sigma_0)\right]^{-\frac{8}{\eta_Z}-1}, \tag{162}$$

and logarithmically increasing $\sigma/M = \sqrt{2/3}\ln(t/t_{\text{kin}})$ one finds for $\eta_Z > 0$ that the contributions $\sim U_E t^2$ in Eq. (156) can no longer be neglected for large enough t . For $\eta_Z < 0$ the field σ would reach σ_0 , and therefore φ diverges, at a finite time t_{end} .

以及对数增长的 $\sigma/M = \sqrt{2/3}\ln(t/t_{\text{kin}})$，可以发现对于 $\eta_Z > 0$，当 t 足够大时，式 (156) 中 $\sim U_E t^2$ 的贡献就无法再忽略了。对 $\eta_Z < 0$，场 σ 会在有限时间 t_{end} 达到 σ_0，因此 φ 发散。

A numerical solution of the cosmological field equations (123) shows that for Z of the order one, the kination epoch is short and the scalar field does not grow to values far beyond the minimum of U . In contrast, for a crossover in Z to values smaller than 1/6 , the kination epoch will stop only due to the presence of radiation and matter. In the absence of radiation and matter, it is a cosmic attractor solution. The scalar potential $\hat{V}$ can decrease to very small values during this epoch.

宇宙场方程 (123) 的数值解表明，当 Z 约为 1 量级时，动力学阶段持续时间很短，标量场的增长不会远超过 U 的极小值。反之，当 Z 交叉到小于 1/6 的取值时，只有存在辐射和物质才会终止动力学阶段。若不存在辐射和物质，这就是一个宇宙吸引子解。在此阶段，标量势 $\hat{V}$ 可以衰减到极小值。

Transition to Radiation and Matter Domination

向辐射主导与物质主导的转变

Assume that after inflation a certain amount of relativistic particles is created. This produces entropy and heats the universe, with particles forming locally an equilibrium state with temperature T . (For a discussion of heating in similar models, see Ref. [108].) The radiation density ρ_r decreases $\sim a^{-4}$, in contrast to the faster decrease of the scalar kinetic energy density $(\partial_t\sigma)^2 \sim t^{-2} \sim a^{-6}$. The kination epoch ends once the radiation energy equals the scalar kinetic energy. If in the following radiation-dominated epoch the scalar potential can be neglected, one has

假设暴胀结束后产生了一定量的相对论性粒子，这会产生熵并加热宇宙，粒子在局部形成温度为 T 的平衡态。(类似模型中的加热过程讨论参见文献 [108]。) 辐射密度 ρ_r 随 $\sim a^{-4}$ 降低，而标量场动能密度 $(\partial_t\sigma)^2 \sim t^{-2} \sim a^{-6}$ 下降得更快，二者形成对比。一旦辐射能量与标量场动能相等，动能主导时期 (kination) 就结束了。若在接下来的辐射主导时期标量势可忽略，则有

$$H = \frac{1}{2t},\ \partial_t^2\sigma + \frac{3}{2t}\partial_t\sigma = 0, \tag{163}$$

with solution

其解为

$$\partial_t\sigma \sim t^{-3/2},\ (\partial_t\sigma)^2 \sim t^{-3} \sim a^{-6}. \tag{164}$$

Thus, the ratio $(\partial_t\sigma)^2/\rho_r \sim a^{-2}$ continues to decrease.

因此，比值 $(\partial_t\sigma)^2/\rho_r \sim a^{-2}$ 持续下降。

During radiation domination the scalar field evolves only slowly

辐射主导阶段，标量场演化十分缓慢

$$\sigma(t) = \sigma_r + 2c_r t_r \left(1 - \sqrt{\frac{t_r}{t}}\right) \approx \sqrt{\frac{8}{3}} M \left(1 - \sqrt{\frac{t_r}{t}}\right), \tag{165}$$

with

满足

$$c_r = (\partial_t \sigma)(t_r) \approx \sqrt{\frac{2}{3}} \frac{M}{t_r},\ \sigma_r = \sigma(t_r), \tag{166}$$

the initial conditions at the onset of radiation domination. The evolution of the scalar field almost stops, approaching $\overline{\sigma} = \sigma_r + 2c_r t_r$. Correspondingly, the scalar potential undergoes only a small change

初始条件设定在辐射主导开端。标量场的演化几乎停止，逐渐趋近 $\overline{\sigma} = \sigma_r + 2c_r t_r$ 。相应地，标量势仅发生微小变化

$$U_E = U_E(t_r) \exp\left[-\sqrt{\frac{8}{3Z}} \left(1 - \sqrt{\frac{t_r}{t}}\right)\right]. \tag{167}$$

The overall picture is simple. The presence of radiation essentially stops the further evolution of the scalar field which settles at the value it has reached at the onset of radiation domination. For a realistic cosmology radiation domination has to set in before nucleosynthesis. Otherwise the different time history due to a substantial kinetic energy of the scalar field would modify the element abundances. For an end of the kination epoch close to nucleosynthesis, small changes of abundances are expected.

整体图像十分清晰: 辐射的存在从根本上停止了标量场的进一步演化，标量场稳定在辐射主导开端时已经达到的取值。对于现实宇宙学，辐射主导必须在核合成开始前确立；否则，标量场可观动能带来的不同时间演化历史会改变元素丰度。若动能主导时期在接近核合成时结束，预计丰度只会发生微小改变。

Due to a baryon asymmetry that has to be created at some moment, radiation domination will be replaced by matter domination. The evolution of the scalar field during matter domination is qualitatively similar to radiation domination. The overall picture is that the evolution of the scalar field becomes very slow such that the kinetic energy $T_E = (\partial_t \sigma)^2/2$ becomes comparable to U_E at some time t_S . The evolution of the scalar field beyond t_S depends on the value of $U_S = U_E(t_S)$. If $\sigma(t_S)$ is in a range where U_S is negative, the gradient term $\sim \partial U_E/\partial\sigma$ in the field equation leads to a subsequent decrease of the scalar field, with U_E becoming more negative as t increases beyond t_S . Cosmology with negative U_E in the present epoch is not comparable with observation. In contrast, for positive U_S the energy density of the scalar field constitutes a form of dynamical dark energy.

由于重子不对称性必须在某一时刻产生，辐射主导最终会被物质主导取代。物质主导阶段标量场的演化在定性上与辐射主导阶段相似。整体图像是: 标量场演化变得极慢，因此动能 $T_E = (\partial_t \sigma)^2/2$ 会在某一时刻 t_S 变得与 U_E 相当。t_S 之后标量场的演化取决于 $U_S = U_E(t_S)$ 的取值。若 $\sigma(t_S)$ 处于 U_S 为负的区间，场方程中的梯度项 $\sim \partial U_E/\partial\sigma$ 会导致标量场随后降低，随着 t 超过 t_S，U_E 变得更负。当前宇宙学中 U_E 为负的模型与观测不符。反之，若 U_S 为正，标量场的能量密度就构成了一种动力学暗能量。

Dynamical Dark Energy

动力学暗能量

Due to the role of the neutrino fluctuations for the scaling solution of $u(\tilde{\rho})$, the potential $U_E(\varphi)$ has a local maximum at positive values, $U_{\max} = U_E(\varphi_{\max}) > 0$. For $U_S > U_{\max}$ the scalar field continues to increase and will reach asymptotically a (approximate) scaling solution [128] for (approximately) constant Z . For $U_S < U_{\max}$ the scalar field typically reaches a turning point and decreases afterwards. Only for U_S sufficiently close to $U_{\max}$, it may reach $U_{\max}$ and turn towards the scaling solution. The precise dynamics of dark energy depends on the ratio $U_S/U_{\max}$ and on Z .

由于中微子涨落对 $u(\tilde{\rho})$ 标度解的作用，势函数 $U_E(\varphi)$ 在正值处存在一个局部极大值，即 $U_{\max} = U_E(\varphi_{\max}) > 0$ 。对于 $U_S > U_{\max}$ ，标量场会持续增长，并渐近趋近于 (近似) 常数 Z 的 (近似) 标度解 [128]。对于 $U_S < U_{\max}$ ，标量场通常会先到达一个拐点，随后开始下降。只有当 U_S 足够接近 $U_{\max}$ 时，标量场才可能到达 $U_{\max}$ ，之后转向标度解。暗能量的精确动力学取决于比值 $U_S/U_{\max}$ 和 Z 。

Cosmological Scaling Solution

宇宙学标度解

For constant Z the scalar field equation reads $\left(\sigma = \sqrt{Z}\varphi, \eta_Z = 0\right)$

对于常数 Z ，标量场方程为 $\left(\sigma = \sqrt{Z}\varphi, \eta_Z = 0\right)$

$$\left(\partial_t^2 + 3H\partial_t\right)\sigma = -\frac{\partial U_E}{\partial\sigma} = \frac{uM^3}{\sqrt{Z}\xi^2}\exp\left(-\frac{\sigma}{\sqrt{Z}M}\right), \tag{168}$$

where

其中

$$U_E = \frac{uM^4}{\xi^2}\exp\left(-\frac{\sigma}{\sqrt{Z}M}\right). \tag{169}$$

The Hubble parameter obeys

哈勃参数满足

$$H^2 = \frac{1}{3M^2}\left[\rho + U_E + \frac{1}{2}(\partial_t\sigma)^2\right], \tag{170}$$

with

且

$$\partial_t\rho + nH\rho = 0. \tag{171}$$

For cosmological scaling solutions [24, 128, 150] the dark energy density follows the same time dependence as the dominant radiation ($n = 4$) or matter ($n = 3$) energy density, with

对于宇宙学标度解 [24, 128, 150]，暗能量密度与主导辐射 ($n = 4$) 或物质 ($n = 3$) 的能量密度具有相同的时间依赖关系，满足

$$\rho = \frac{\rho_0 M^2}{t^2},\ H = \frac{2}{nt}. \tag{172}$$

The scalar field changes logarithmically

标量场作对数变化

$$\sigma = \sigma_0 + c_\sigma M \ln\left(\frac{t}{t_0}\right), \tag{173}$$

such that Eq. (168) becomes

因此式 (168) 变为

$$\left(\frac{6}{n} - 1\right)c_\sigma t^{-2} = \frac{uM^2}{\sqrt{Z}\xi^2}\exp\left(-\frac{\sigma_0}{\sqrt{Z}M}\right)\left(\frac{t}{t_0}\right)^{-\frac{c_\sigma}{\sqrt{Z}}}. \tag{174}$$

For positive u one has $c_\sigma > 0$, such that Eq. (174) is obeyed for

当 u 为正时，有 $c_\sigma > 0$ ，因此式 (174) 在下述条件下成立

$$c_\sigma = 2\sqrt{Z} \tag{175}$$

and

且

$$\frac{uM^2t_0^2}{\xi^2}\exp\left(-\frac{\sigma_0}{\sqrt{Z}M}\right) = 2\left(\frac{6}{n} - 1\right)Z. \tag{176}$$

From Eq. (170) one infers

由式 (170) 可得

$$\rho_0 = \frac{12}{n}\left(\frac{1}{n} - Z\right). \tag{177}$$

The fraction of homogeneous dark energy is given by

均匀暗能量占比由下式给出

$$\Omega_h = \frac{U_E + \frac{1}{2}(\partial_t \sigma)^2}{3M^2H^2} = Zn. \tag{178}$$

The scaling solution is a cosmic attractor solution in the sense that neighboring solutions approach it for increasing t [128, 150] . For negative u no scaling solution of this type exists.

随着 t [128, 150] 增大，邻域解会趋近该标度解，因此标度解是宇宙吸引子解。当 u 为负时，不存在该类型的标度解。

Quintessence

精质

Quantum gravity predictions for dynamical dark energy or quintessence are only partial. The scaling solution predicts the potential $u(\tilde{\rho})$ rather accurately. On the other hand, the translation to $U_E(\sigma)$ involves the curvature coefficient $f(\tilde{\rho})$ and the wave function renormalization $Z(\tilde{\rho})$. Even in the approximation where we only employ two constants ξ_∞ and Z , with $f(\tilde{\rho} \gg 1) = 2\xi_\infty\tilde{\rho}$, the dynamics of quintessence further depends on U_S or, equivalently, σ_S . We may interpret these values in terms of "initial conditions" for the evolution in the present epoch. In principle, σ_S is computable if $u(\tilde{\rho})$, $f(\tilde{\rho})$, and $K(\tilde{\rho})$ are known. The duration of the kination epoch depends on the detailed dynamics near the end of inflation. For given u, f, K inflation is an attractor solution which has no free parameters. All quantities are determined as functions of the number of e -foldings before the end of inflation on which the dynamics depends. In practice, however, σ_S depends on too many details. We may take the inverse attitude and consider σ_S/M as a free dimensionless parameter. If a value of σ_S leads to realistic cosmology, we may use this as a constraint on the detailed physics near the end of inflation. In this sense σ_S "monitors" the quantum fluctuations at scales relevant for the end of inflation.

量子引力对动态暗能量即精质的预言仍是不完全的。标度解相当准确地预言了势函数 $u(\tilde{\rho})$ 。另一方面，转换到 $U_E(\sigma)$ 需要用到曲率系数 $f(\tilde{\rho})$ 和波函数重整化 $Z(\tilde{\rho})$ 。即使在我们仅使用两个常数 ξ_∞ 和 Z (满足 $f(\tilde{\rho} \gg 1) = 2\xi_\infty\tilde{\rho}$) 的近似下，精质的动力学还进一步依赖 U_S ，或者等价地说，依赖 σ_S 。我们可以将这些值解释为当前宇宙演化的“初始条件”。原则上，只要已知 $u(\tilde{\rho})$, $f(\tilde{\rho})$ 和 $K(\tilde{\rho})$ ，就可以计算出 σ_S 。动能时期的持续时间取决于暴胀结束附近的详细动力学。对于给定的 u, f, K ，暴胀是没有自由参数的吸引子解。所有物理量都由暴胀结束前 e 个暴胀折叠数的函数确定，动力学依赖于该参数。但实际上，σ_S 依赖的细节过多。我们可以反过来将 σ_S/M 视为一个自由无量纲参数。如果某一 σ_S 的取值能得到符合实际的宇宙学结果，我们就可以将其用作暴胀结束附近详细物理的约束。从这个意义上说，σ_S “监测着”与暴胀结束相关尺度上的量子涨落。

The value of the potential at the maximum $U_{\max}$ is determined for $f = 2\xi\tilde{\rho}$ by the zero of c_U ,

势函数在最大值 $U_{\max}$ 处的取值，对于 $f = 2\xi\tilde{\rho}$ 由 c_U 的零点确定,

$$\tilde{\rho}\frac{\partial\widehat{V}}{\partial\tilde{\rho}} = \frac{1}{f^2}\left(\tilde{\rho}\partial_{\tilde{\rho}}u - 2\right) = -\frac{2c_U}{f^2} = 0. \tag{179}$$

According to Eq. (100), this is given by $\left(h_v^2 = \xi m_v^2/M^2\right)$

根据式 (100)，它由 $\left(h_v^2 = \xi m_v^2/M^2\right)$ 给出

$$2h_v^2\tilde{\rho}_{\max} = \frac{1}{5},\ \tilde{\rho}_{\max} = \frac{M^2}{10\xi m_v^2}, \tag{180}$$

with

其中

$$M^4 \exp\left(-\frac{\varphi_{\max}}{M}\right) = 25\xi^2 m_v^4. \tag{181}$$

The maximum value of the potential is given by the neutrino mass

势函数的最大值由中微子质量给出

$$U_{\max} = 25u\left(\tilde{\rho}_{\max}\right) m_v^4, \tag{182}$$

where

其中

$$u\left(\tilde{\rho}_{\max}\right) = \frac{1}{128\pi^2}\left(5 - 6t_u\left(0.2\right)\right) = \frac{0.54}{128\pi^2} = 4.27 \cdot 10^{-4}, \tag{183}$$

such that

满足

$$\frac{U_{\max}^{1/4}}{m_v} = 0.32 \tag{184}$$

It is striking that the characteristic scale for dynamical dark energy turns out very close to the neutrino mass, with details depending on the precise mass pattern for the three neutrinos. On the other hand, this makes the detailed understanding of the quintessence dynamics more involved. We observe that a constant value $\sigma(t) = \sigma_{\max}$ is an exact solution, with effective cosmological constant given by $U_{\max}$ not very far from the observed dark energy density. For fundamental scale invariance the solution $\sigma = \sigma_{\max}$ is the only solution for which dark energy is a constant. The generic prediction is a dynamical form of dark energy rather than an effective cosmological constant. For cosmologies in the vicinity of this solution, one may expect a slow evolution. Further, complexity may arise if a crossover in the neutrino sector induces in the Einstein frame a σ -dependence of the neutrino masses, as for growing neutrino quintessence [5, 7, 17, 83, 132] . Within the scaling solution this happens if h_υ depends on $\tilde{\rho}$. In this case a possible cosmic scaling solution ends once the neutrinos become nonrelativistic.

值得注意的是,动态暗能量的特征尺度被发现非常接近中微子质量,具体细节取决于三种中微子的精确质量模式。但另一方面, 这也让对精质动力学的详细理解变得更复杂。我们发现, 常数 $\sigma(t) = \sigma_{\max}$ 是一个精确解，其有效宇宙学常数 $U_{\max}$ 离观测到的暗能量密度并不远。对于基本标度不变性，解 $\sigma = \sigma_{\max}$ 是唯一能让暗能量保持恒定的解。一般的预言是暗能量为动态形式，而非有效宇宙学常数。在靠近该解的宇宙学中，暗能量会发生慢演化。此外，如果中微子 sector 的突变在爱因斯坦框架中诱导出中微子质量对 σ 的依赖，就像增长中微子精质 [5, 7, 17, 83, 132] 那样，问题会变得更复杂。在标度解框架内，当 h_υ 依赖 $\tilde{\rho}$ 时就会出现这种情况。这种情况下，一旦中微子变成非相对论性的，原有的宇宙标度解就会终止。

Early Dark Energy

早期暗能量

Fundamental scale invariance has an important consequence for the possible existence of early dark energy. For the epochs of radiation and matter domination, the scalar potential is bounded by $U_{\max}$, typically with U_S having a similar order of magnitude as $U_{\max}$. For the epochs of radiation-matter equality or last scattering, the energy density ρ_E in radiation and matter obeys $\rho_E \leq 0.2\mathrm{eV}$. This imposes for these epochs a bound on the fraction of early dark energy Ω_{EDE} ,

基础尺度不变性对早期暗能量的可能存在具有重要意义。在辐射和物质主导时期，标量势受 $U_{\max}$ 限制，通常 U_S 与 $U_{\max}$ 量级相近。在辐射-物质相等或最后散射时期，辐射和物质中的能量密度 ρ_E 满足 $\rho_E \leq 0.2\mathrm{eV}$ 。这为上述时期的早期暗能量占比 Ω_{EDE} 给出了限制,

$$\Omega_{\mathrm{EDE}} \leq \frac{U_{\max}}{\rho_E} \approx 6.5\left(\frac{m_\upsilon}{\mathrm{eV}}\right)^4. \tag{185}$$

During these epochs the kinetic energy of the scalar field is already tiny. According to Eq. (164) its relative fraction has decreased by a factor $\sim (a_{\mathrm{NS}}/a)^2$ since nucleosynthesis.

在上述时期内，标量场的动能已经非常小。根据式 (164)，自核合成以来，其相对占比已经降低了 $\sim (a_{\mathrm{NS}}/a)^2$ 倍。

Fundamental Scale Invariance

基本标度不变性

Fundamental scale invariance [146] states that the world is described by the exact scaling solution of functional flow equations for quantum gravity. This property follows for a theory without any intrinsic length or mass scale. A theory with fundamental scale invariance can be formulated entirely in terms of ”scale-invariant fields” without any appearance of k . This includes the existence of a continuum limit. In consequence, the quantum effective action does not involve k once expressed in terms of the scale-invariant fields. In our context the scale-invariant fields are $\widetilde{\chi} = \chi/k$ and $\tilde{g}_{\mu\nu} = k^2 g_{\mu\nu}$. Indeed, the use of $\tilde{g}_{\mu\nu}$ absorbs the factors k^4 in U and k^2 in F in Eq. (3). Expressing the scaling solution in terms of scale-invariant fields, the effective action no longer involves k :

基本标度不变性 [146] 指出，世界由量子引力泛函流方程的精确标度解描述。该性质适用于不存在任何内禀长度或质量标度的理论。具有基本标度不变性的理论可以完全通过“标度不变场”构造，不会出现 k 。这包含连续极限的存在性。因此，当用标度不变场表述时，量子有效作用量不涉及 k 。在本文的语境中，标度不变场为 $\widetilde{\chi} = \chi/k$ 和 $\tilde{g}_{\mu\nu} = k^2 g_{\mu\nu}$ 。事实上，采用 $\tilde{g}_{\mu\nu}$ 会吸收式 (3) 中 U 里的因子 k^4 和 F 里的因子 k^2 。将标度解用标度不变场表述后，有效作用量不再包含 k :

$$\Gamma = \int_x \sqrt{\tilde{g}} \left\{ -\frac{f(\tilde{\rho})}{2} \widetilde{R} + \frac{1}{2} K(\tilde{\rho}) \partial_\mu \widetilde{\chi} \partial_\nu \widetilde{\chi} \tilde{g}^{\mu\nu} + u(\tilde{\rho}) \right\}. \tag{186}$$

For computing the scaling functions $u(\tilde{\rho}), f(\tilde{\rho})$, or $K(\tilde{\rho})$ for a theory with fundamental scale invariance, we have employed functional flow equations for the variation of an effective infrared cutoff R_k . This concept can be employed for a formulation in terms of scale-invariant fields as well [146]. The flow equation (50) formulates the k -dependence of an effective action for which only fluctuations with squared momenta $q^2 > k^2$ are included. It is formulated for fixed fields $\chi, g_{\mu\nu}$ such that the infrared cutoff requires for the momenta of fluctuations that are effectively included in $\Gamma_k \bar{q}^2 = q_\mu q_\nu g^{\mu\nu}/\chi^2 \gtrsim k^2/\chi^2$. (Roughly speaking the dimensionless quantity $\bar{q}^2$ is the squared momentum in units of the χ -dependent masses.) In terms of the scale-invariant fields, this condition becomes $\bar{q}^2 \gtrsim \widetilde{\chi}^{-2}$. No scale k appears anymore, while the fluctuation effects are not studied for varying field values $\widetilde{\chi}$. The flow equation describes how the effective action changes as additional fluctuations are included due to changing dimensionless masses $\sim \widetilde{\chi}$. This is the content of the differential equations (81) for the $\tilde{\rho}$ -dependence of the scaling solutions.

为计算具有基本标度不变性理论的标度函数 $u(\tilde{\rho}), f(\tilde{\rho})$ 或 $K(\tilde{\rho})$ ，我们采用泛函流方程描述有效红外截断 R_k 的变化。该概念同样可用于标度不变场的表述 [146]。流方程 (50) 给出了有效作用量对仅包含平方动量 $q^2 > k^2$ 涨落的项的 k 依赖关系。它是针对固定场 $\chi, g_{\mu\nu}$ 表述的，因此红外截断对有效纳入 $\Gamma_k \bar{q}^2 = q_\mu q_\nu g^{\mu\nu}/\chi^2 \gtrsim k^2/\chi^2$ 的涨落的动量提出了要求。(粗略来说，无量纲量 $\bar{q}^2$ 是以依赖 χ 的质量为单位的平方动量。) 用标度不变场表述时，该条件变为 $\bar{q}^2 \gtrsim \widetilde{\chi}^{-2}$ 。不再出现标度 k ，同时也不对变化的场值 $\widetilde{\chi}$ 研究涨落效应。流方程描述了当无量纲质量 $\sim \widetilde{\chi}$ 变化导致额外涨落被纳入时，有效作用量的变化方式。这就是微分方程 (81) 中关于标度解 $\tilde{\rho}$ 依赖关系的核心内容。

Fundamental scale invariance requires the existence of a scaling solution. In the other direction, the existence of a scaling solution guarantees the existence of an effective action which is compatible with fundamental scale invariance. We have already observed that for the scaling solution the scale k is no longer present if we formulate the field equations in the Einstein frame. In view of the scale-invariant formulation (186), this should not be a surprise.

基本标度不变性要求存在标度解。反过来，标度解的存在保证了存在与基本标度不变性相容的有效作用量。我们已经注意到，若在爱因斯坦框架中表述场方程，对标度解而言标度 k 已不再存在。从标度不变表述 (186) 来看，这并不意外。

Predictivity

可预测性

Theories with fundamental scale invariance have a high predictive power. As compared to general renormalizable quantum field theories formulated in terms of a UV fixed point, the relevant parameters for the flow away from the scaling solution are absent. This yields important additional restrictions in the space of all possible renormalizable quantum field theories. Free parameters arise only if there exists a whole family of scaling solutions that can be parameterized by these parameters.

具备基本标度不变性的理论拥有很强的预测能力。与以紫外不动点为基础表述的普通可重整化量子场论相比，不存在用于描述标度解之外流的相关参数，这为所有可能的可重整化量子场论的参数空间带来了重要的额外限制。只有当存在一整个可由这些参数参数化的标度解族时，才会产生自由参数。

In practical terms the restrictions arise because the scaling solutions have to exist for the whole range of fields, momenta, etc. Concerning our truncation the scaling solutions for u, f , and K have to exist for the whole range of $\tilde{\rho}$ from zero to infinity. Properties at both ends of the interval $0 \leq \tilde{\rho} < \infty$ matter for the existence of solutions, somewhat similar to the possible solutions of the Schrödinger equation for radial wave functions in the hydrogen atom. For the region of small $\tilde{\rho}$, we may consider potential scaling solutions of Eq. (81) as an initial value problem, with initial data set at $\tilde{\rho} = 0$. Not all of the initial data lead to solutions that can be extended to the whole range of $\tilde{\rho}$. Furthermore, we know that for $\tilde{\rho} \to \infty$ the function $u(\tilde{\rho})$ has to approach a constant u_∞ , and $f(\tilde{\rho})/\tilde{\rho}$ has to reach a constant value $2\xi_\infty$.

实际上，这些限制来源于标度解必须存在于全部的场、动量等范围内。就我们的截断而言，u, f 和 K 的标度解必须存在于从零到无穷的整个 $\tilde{\rho}$ 范围内。区间两端 $0 \leq \tilde{\rho} < \infty$ 的性质都会影响解的存在性，这和氢原子径向波函数薛定谔方程的解有一定相似性。对于小 $\tilde{\rho}$ 区域，我们可以将式 (81) 的潜在标度解作为初值问题处理，在 $\tilde{\rho} = 0$ 处给定初始数据。并非所有初始数据都能得到可延拓至整个 $\tilde{\rho}$ 范围的解。此外我们已知，当 $\tilde{\rho} \to \infty$ 时，函数 $u(\tilde{\rho})$ 必须趋近于常数 u_∞ ，$f(\tilde{\rho})/\tilde{\rho}$ 必须达到常数值 $2\xi_\infty$ 。

These conditions put severe restrictions on the scaling solutions for models with a given content of particles. This holds, in particular, for the range of large $\tilde{\rho}$ for which the particle content is restricted by observation of the "low-energy physics." For the example of the standard model, the scaling solution puts an upper bound $U_{\max}$ for the potential U_E in this region. For three degenerate neutrinos, it is given by Eq. (184), with calculable modifications for arbitrary masses of the neutrinos. It also predicts for the standard model coupled to gravity that U_E remains negative for the whole range of $\tilde{\rho}$ below a value close to the maximum of U_E . The minimum of U_E occurs in this case at $\tilde{\rho} = 0$ for negative U_E . It is difficult to see how realistic cosmology emerges in this case within the truncation (3). In contrast, for grand unified theories, $\tilde{\rho} = 0$ corresponds to the maximum of U_E , predicting an inflationary epoch. The restrictions for the UV fixed points of $u(0)$ and $f(0)$ remain valid for arbitrary renormalizable theories since these are also the UV fixed point values for the flow with k . For $k \to 0$ these restrictions are typically no longer present for general renormalizable theories, however. They can be circumvented, at least partially, by the flow away from the scaling solution.

这些条件给给定粒子内容模型的标度解带来了严格限制，尤其对大 $\tilde{\rho}$ 范围成立，该范围内的粒子内容受到“低能物理”观测的约束。以标准模型为例，标度解为该区域的势 U_E 给出了一个上界 $U_{\max}$ 。对于三个简并中微子，该上界由式 (184) 给出，任意中微子质量仅带来可计算的修正。该模型还预测，在耦合引力的标准模型中，U_E 在整个 $\tilde{\rho}$ 范围内始终保持负值，直到接近 U_E 最大值的位置。这种情况下 U_E 的最小值出现在 $\tilde{\rho} = 0$ 处，对应 U_E 为负，很难看出该情形下截断 (3) 如何得到现实的宇宙学。与之相对，大统一理论中 $\tilde{\rho} = 0$ 对应 U_E 的最大值，预测了一个暴胀时期。对任意可重整化理论，$u(0)$ 和 $f(0)$ 紫外不动点受到的限制仍然成立，因为这些也是带 k 流的紫外不动点值。不过对于一般可重整化理论，$k \to 0$ 通常不再受这些限制，偏离标度解的流至少可以部分绕过这些限制。

Solution of Cosmological Constant Problem

宇宙学常数问题的解

A central reason why we focus this work on the scaling solution is the possible dynamical solution of the cosmological constant problem [125] without invoking any small dimensionless parameter. This distinguishes the scaling solution or close neighbors of it from the more general solutions of flow equations for quantum gravity.

我们将这项工作的重点放在标度解上的一个核心原因是，它无需引入任何小无量纲参数，就可以为宇宙学常数问题提供动力学解 [125]。这一点将标度解或其邻近解与量子引力流方程的更通用解区分开来。

The central ingredient for this dynamical solution is the decrease of $\hat{V}(\tilde{\rho})$ to zero for $\tilde{\rho} \to \infty$. If the cosmological dynamics drives $\tilde{\rho}$ towards infinity in the infinite future, the potential in the Einstein frame $U_E = \hat{V} M^4$ vanishes for $t \to \infty$. The cosmological constant is driven to zero dynamically. More precisely, only the dimensionless ratio of the scalar potential U over the fourth power of the Planck mass F^2 is observable. This ratio vanishes due to $U(\rho \to \infty) \sim k^4, F(\rho \to \infty) \sim \rho$, such that $\hat{V} = U/F^4 \sim \rho^{-2}$. For the scaling solution this is directly visible in the relation

该动力学解的核心要素是，当 $\tilde{\rho} \to \infty$ 时 $\hat{V}(\tilde{\rho})$ 减小至零。如果宇宙动力学在无限未来驱动 $\tilde{\rho}$ 趋于无穷，那么爱因斯坦框架中的势 $U_E = \hat{V} M^4$ 在 $t \to \infty$ 时消失。宇宙学常数被动力学驱动至零。更准确地说，只有标量势 U 与普朗克质量四次方 F^2 的无量纲比值是可观测的。由于 $U(\rho \to \infty) \sim k^4, F(\rho \to \infty) \sim \rho$ ，该比值消失，因此得到 $\hat{V} = U/F^4 \sim \rho^{-2}$ 。对于标度解，这一点可以在下述关系中直接看出

$$\hat{V}(\tilde{\rho} \to \infty) = \frac{u}{f^2}(\tilde{\rho} \to \infty) = \frac{u_\infty^2}{4\xi_\infty^2 \tilde{\rho}^2}. \tag{187}$$

For the scaling solution the only mass scale is given by the renormalization scale k . The large present value of $\tilde{\rho}$ is only due to the evolution of the universe - in Planck units the present universe is very old. This large value explains the tiny value of the present dark energy density in Planck units, $\hat{V}(\text{today}) \approx 10^{-120}$. No small parameter or small ratio of parameters is invoked, similar to the present small value of the matter and radiation energy density in Planck units, which also results from the large age of the universe.

对标度解而言，唯一的质量尺度由重整化标度 k 给出。$\tilde{\rho}$ 当前的大数值仅源于宇宙的演化——以普朗克单位衡量，当前宇宙已经非常古老。这个大数值解释了普朗克单位下当前暗能量密度为什么极小，即 $\hat{V}$(今天) $\approx 10^{-120}$ 。我们不需要引入任何小参数或小参数比，这和普朗克单位下当前物质与辐射能量密度数值很小类似，后者同样源于宇宙的年龄很大。

Relevant Parameters

相关参数

In order to appreciate the role of fundamental scale invariance for the dynamical solution of the cosmological constant problem, we compare with general renormal-izable quantum gravity. Indeed, the situation is different for general solutions of the flow equation with relevant parameters. Typically, both the Planck mass and the cosmological constant correspond to relevant parameters. The general solution of the flow equation takes then the qualitative form

为了理解基本标度不变性对宇宙学常数问题动力学解的作用，我们将其与一般可重整量子引力进行对比。实际上，对于带有相关参数的流方程通解，情况有所不同。普朗克质量和宇宙学常数通常都对应相关参数，此时流方程通解的定性形式为

$$F = f(\tilde{\rho})\,k^2 + \mu_p^2 \approx f_0 k^2 + \xi_\infty \chi^2 + \mu_p^2,$$

$$U = u(\tilde{\rho})\,k^4 + \lambda, \tag{188}$$

where both μ_p^2 and λ set intrinsic mass scales related to relevant parameters. These are free parameters of the model. (More generally, μ_p^2 and λ may be functions of χ which only depend on two constants that we associate here to the constants μ_p^2 and λ . The computation of these functions depends on the flow in a range of k where deviations from the scaling solution grow large.)

其中 μ_p^2 和 λ 都设定了与相关参数相关的内禀质量尺度，二者是模型的自由参数。(更一般地，μ_p^2 和 λ 可以是 χ 的函数，仅依赖两个常数，此处我们将这两个常数对应为常数 μ_p^2 和 λ 。这些函数的计算依赖于 k 范围内的流，在该范围内对标度解的偏离会变得很大。)

If the present value of F is dominated by μ_p^2 , i.e., $\xi\chi^2 \lesssim \mu_p^2$, one finds $\hat{V} \approx \lambda/\mu_p^4$. A small value of $\hat{V}$ corresponds now to a ratio of parameters that has to assume the tiny value 10^{-120} . This requires "tuning" of the relevant parameters. This situation is also realized in the absence of a scalar field χ , or if χ settles in the cosmological evolution to a constant value $\overline{\chi} = c\mu_p$. In the latter case one replaces in Eq. (188) μ_p^2 by $\mu_p^2\left(1+\xi c^2\right)$.

如果 F 的当前值由 μ_p^2 主导，即 $\xi\chi^2 \lesssim \mu_p^2$ ， 可得 $\hat{V} \approx \lambda/\mu_p^4$ 。$\hat{V}$ 的小值对应参数比必须取极小值 10^{-120} ，这就要求对相关参数进行“微调”。这种情况在不存在标量场 χ ，或 χ 在宇宙演化中稳定为常数 $\overline{\chi} = c\mu_p$ 时也会成立。后者的情况需要将式 (188) 中 μ_p^2 替换为 $\mu_p^2\left(1+\xi c^2\right)$ 。

Without a tuning of relevant parameters, i.e., for $\lambda \approx \mu_p^4$, a dynamical solution of the cosmological constant problem remains still possible for crossover cosmologies leading to a present value of $\chi \approx 10^{30}\mu_p$.

As compared to the scaling solution one replaces $f_0 k^2 \to f_0 k^2 + \mu_p^2, uk^4 \to uk^4 + \lambda$. This is the case discussed in section "Quantum Gravity" for which the scaling solution becomes relevant only for $k^2 \gtrsim \mu_p^2$. Indeed, in this range of k we can neglect μ_p^2 and λ in Eq. (188). While the relevant parameters μ_p^2 and λ become unimportant in the UV limit $k \to \infty$, they still influence the early cosmology for small values of the scalar field χ . The reason is that the limits $k \to \infty$ and $\chi \to 0$ can no longer be identified as for the exact scaling solution.

若不对相关参数做微调，即对于 $\lambda \approx \mu_p^4$，宇宙学常数问题仍可通过穿越宇宙学得到动力学解，使得 $\chi \approx 10^{30}\mu_p$ 取当前值。与标度解相比，我们替换了 $f_0 k^2 \to f_0 k^2 + \mu_p^2, uk^4 \to uk^4 + \lambda$。这正是"量子引力"章节讨论的情况，在该情况中标度解仅对 $k^2 \gtrsim \mu_p^2$ relevant。实际上，在 k 的这个范围内，我们可以忽略式 (188) 中的 μ_p^2 和 λ。尽管相关参数 μ_p^2 和 λ 在紫外极限 $k \to \infty$ 下变得无关，但它们仍会影响标量场 χ 取较小值时的早期宇宙学。原因在于，极限 $k \to \infty$ 和 $\chi \to 0$ 无法再像精确标度解那样被等同起来。

In the presence of relevant parameters, the effective scalar potential in the Einstein frame becomes

存在相关参数时，爱因斯坦框架下的有效标量势变为

$$U_E = \frac{\left(u(\tilde{\rho})\, k^4 + \lambda\right) M^4}{\left(f_0 k^2 + 2\xi_\infty \tilde{\rho} + \mu_p^2\right)^2}, \tag{189}$$

with $u(\tilde{\rho})$ the scaling potential. This potential can be positive for $\tilde{\rho} = 0$ even for $u_0 < 0$, provided $\lambda > -u_0 k^4$. In the presence of relevant parameters, an inflationary epoch may become possible for the standard model of particle physics coupled to gravity. On the way towards $k \to 0$, our approximation at $\tilde{\rho} = 0$ breaks down since $v < 1$ requires $2\left(\lambda + u_0 k^4\right) < \mu_p^2 k^2 + f_0 k^4$. For an inflationary epoch the scale k may, however, be replaced by an effective geometrical cutoff.

其中 $u(\tilde{\rho})$ 为标度势。只要满足 $\lambda > -u_0 k^4$，即使在 $u_0 < 0$ 的条件下，该势在 $\tilde{\rho} = 0$ 处也可以为正。存在相关参数时，引力耦合粒子物理标准模型也可能发生暴胀时期。在趋近 $k \to 0$ 的过程中，我们在 $\tilde{\rho} = 0$ 处的近似会失效，因为 $v < 1$ 要求 $2\left(\lambda + u_0 k^4\right) < \mu_p^2 k^2 + f_0 k^4$。但对于暴胀时期，标度 k 可以被有效几何截断取代。

If $u(\tilde{\rho}) + \lambda/k^4$ is positive for the whole range of $\tilde{\rho}$, a crossover cosmology with χ diverging in the infinite future is rather generic. The potential $U_E(\varphi) = M^4 u/f^2$ typically decreases monotonically for increasing φ due to the exponential increase of f . For such a potential φ increases monotonically with time, at least asymptotically. In the infinite future one reaches $U_E(\varphi \to \infty) = 0$. This situation is realized for the scaling solution of pure gravity coupled to the scalar field χ . For particle physics with the standard model as an effective low-energy theory, the situation is more complex. For fundamental scale invariance the potential U_E is now negative for a substantial range of χ . The asymptotic cosmological scaling solution (173) is now only reached if the increase of χ during the kination epoch is large enough. This poses conditions on $Z(\varphi)$. This issue can be avoided for the more general solution of the flow equations (188) provided that $\lambda + k^4 u(\tilde{\rho})$ is positive for the whole range of $\tilde{\rho}$. The price to pay is a partial loss of predictive power.

若 $u(\tilde{\rho})+\lambda/k^4$ 在 $\tilde{\rho}$ 的整个取值范围内为正，那么带有 χ 在无限未来发散的交叉宇宙学是相当普遍的。由于 f 呈指数增长，势函数 $U_E(\varphi)= M^4u/f^2$ 通常会随 φ 增大而单调递减。对于这类势函数，φ 至少在渐近区域会随时间单调递增。在无限未来会演化到 $U_E(\varphi\to\infty)=0$ 。这种情况出现在耦合标量场 χ 的纯引力标度解中。对于以标准模型为低能有效理论的粒子物理，情况则更为复杂。在基本标度不变性框架下，势函数 U_E 在 χ 的很大取值范围内均为负值。只有当动力学演化时期 χ 的增长足够大时，才能最终达到渐近宇宙学标度解 (173)。这对 $Z(\varphi)$ 提出了限制条件。若 $\lambda+k^4u(\tilde{\rho})$ 在 $\tilde{\rho}$ 的整个取值范围内为正，那么流方程 (188) 的更一般解可以避开这个问题，代价是部分丧失了预言能力。

We conclude that the basic ingredients for a dynamical solution of the cosmological constant problem are similar for general renormalizable quantum gravity without fine-tuning of parameters (i.e., for λ of the order μ_p^4) and for fundamental scale invariance. This requires, however, that the scale μ_p set by the relevant parameters is much smaller than the present value of the Planck mass given by χ , typically $\mu_p\approx 10^{-2}\mathrm{eV}$. In both cases late cosmology corresponds to large χ such that F is dominated by $\xi_\infty\chi^2$. On the other hand, U becomes a constant U_∞ for large χ , resulting in $U_E\sim U_\infty M^4/\left(\xi_\infty^2\chi^4\right)$. As a result, U_E vanishes as χ increases towards infinity. For cosmology of late stages of inflation and all later epochs, only the relevant parameter λ distinguishes general renormalizable gravity from the setting of fundamental scale invariance.

我们得出结论: 对于无需参数精细调节的一般可重整量子引力 (即 λ 量级为 μ_p^4) 和基本标度不变性来说，宇宙学常数问题动力学解的基本构成是相似的。但这要求相关参数设定的标度 μ_p 远小于由 χ 给出的当前普朗克质量，通常满足 $\mu_p\approx 10^{-2}\mathrm{eV}$ 。两种情况下，晚期宇宙学都对应大的 χ ，此时 F 由 $\xi_\infty\chi^2$ 主导。另一方面，当 χ 很大时，U 变为常数 U_∞ ，从而得到 $U_E\sim U_\infty M^4/\left(\xi_\infty^2\chi^4\right)$。因此，随着 χ 趋于无穷大，U_E 逐渐消失。对于暴胀晚期及之后所有宇宙学演化阶段，只有相关参数 λ 能区分一般可重整引力与基本标度不变性框架。

Discussion

讨论

A formulation of quantum gravity as a quantum field theory for the metric, together with the power of modern functional renormalization group techniques to compute the effects of quantum fluctuations of the metric, yields a predictive scheme for cosmology. Important functions as the inflaton potential for early cosmology, or the cosmon (quintessence) potential for late cosmology, can no longer be chosen completely freely. The scaling solution which is necessary to render the quantum field theory renormalizable imposes important restrictions on the shape of these potentials. At low energies and for small field values, renormalizability enforces for the Higgs scalar and other scalars with gauge or Yukawa interactions to the fields of the standard model an approximately polynomial potential. This is not the case for the scalar singlet discussed in this work. The scaling solution predicts a potential that deviates strongly from a polynomial form. The potential in the Einstein frame, or the frame-invariant potential, approaches zero exponentially for large positive values of the canonical scalar field σ , while it tends to a positive constant in the opposite limit of large negative σ .

将量子引力表述为度规的量子场论，结合现代泛函重整化群技术计算度规量子涨落效应的强大能力，为宇宙学构建了一套可预言的框架。早期宇宙学的暴胀子势、晚期宇宙学的宇宙子 (精质) 势等重要函数不再能被完全自由选取。让量子场论可重整化所必需的标度解对这些势的形状施加了重要限制。在低能和小场值情况下，可重整化性要求与标准模型场存在规范或汤川相互作用的希格斯标量及其他标量具有近似多项式势。本文讨论的标量单态并非如此。标度解预言的势与多项式形式偏差显著。对于正则标量场 σ 的大正取值，爱因斯坦框架下的势或框架不变势会指数趋近于零，而在 σ 大负取值的相反极限下，势趋于一个正常数。

It is impressive to see how from these simple properties an overall cosmology with rather realistic features emerges. Cosmology describes a crossover from a UV fixed point, realized for $\sigma \to -\infty$ in the infinite past, to an IR fixed point for $\sigma \to \infty$ in the infinite future. In between, the sequence of characteristic cosmological epochs finds its place: inflation, kination, radiation domination, matter domination, and dark energy domination.

值得关注的是，从这些简单性质中竟涌现出了一套整体特征相当符合实际的宇宙学。宇宙学描述了从无穷过去 $\sigma \to -\infty$ 对应的紫外不动点，到无穷未来 $\sigma \to \infty$ 对应的红外不动点的渡越过程。在这两个不动点之间，依次排列着各个特征宇宙学时期: 暴胀、动能主导、辐射主导、物质主导和暗能量主导。

The rather detailed information about the scaling solution yields further predictions for the case of fundamental scale invariance.

关于标度解的详细信息还为基础标度不变性情形给出了更多预言。

(i) Within the truncation with up to two derivatives, the standard model coupled to quantum gravity is not viable. This is related to the negative value of the scaling potential at $\tilde{\rho} = 0, u_0 < 0$. The conclusion remains the same if we replace the scalar singlet by the Higgs doublet, as for Higgs inflation. A viable quantum gravity extension of the standard model requires an important role of higher-derivative terms [58]. We pursue here the alternative of an extension to a grand unified symmetry. This implies positive u_0 for which an inflationary epoch is predicted.

(i) 在导数最高为二阶的截断下，耦合量子引力的标准模型并不自洽，这与 $\tilde{\rho} = 0, u_0 < 0$ 处标度势为负值有关。即使我们像希格斯暴胀那样将标量单态替换为希格斯二重态，结论依然成立。标准模型要得到自洽的量子引力扩展，就必须让高阶导数项发挥重要作用 [58]。本文研究的是另一种方案: 将模型扩展为大统一对称性。这会得到正的 u_0 ，并预言存在一个暴胀时期。

(ii) Inflation ends at the latest at the transition where many bosons beyond the standard model particles become massive. This is typically the GUT-phase transition in grand unified theories. An end of inflation near the GUT-phase transition can naturally explain the small amplitude of the primordial fluctuations.

(ii) 暴胀最晚在超出标准模型的大量玻色子获得质量的相变处结束，这在大统一理论中通常就是大统一相变。暴胀在大统一相变附近结束可以自然解释原初涨落的小振幅。

(iii) Dark energy is dynamical rather than being a cosmological constant.

(iii) 暗能量是动力学的，而非宇宙学常数。

(iv) For the range of the scalar field relevant after inflation, the potential is bounded by a maximal value $U_{\max} = (0.32m_{\upsilon})^4$, or similar for a nondegenerate mass pattern of the three neutrinos. The neutrino mass m_{υ} sets the characteristic scale for the dynamics of dark energy. The maximal potential $U_{\max}$ limits the amount of a possible early dark energy [35-37,61,62,87,101,131,157] unless additional degrees of freedom are introduced.

(iv) 对于暴胀后相关的标量场范围，势被最大值 $U_{\max} = (0.32m_{\upsilon})^4$ 限制，对于三种中微子非简并质量谱的情况也类似。中微子质量 m_{υ} 设定了暗能量动力学的特征标度。除非引入额外自由度，否则最大势 $U_{\max}$ 会限制早期暗能量的可能占比 [35-37,61,62,87,101,131,157]。

So far we have not used quantitative information about the wave function renormalization or kinetial $Z(\varphi)$ or $K(\chi)$. Once the scaling solution is computed for this function, important additional predictions become possible. For example, there is an explicit formula for the slow-roll coefficients ε and η during inflation in terms of the three scaling functions. Since these coefficients determine directly the spectrum of primordial cosmic fluctuations, such a prediction is testable. It is remarkable that cosmological observations will be able to falsify a model of fundamental scale invariance with a single scalar field. Future computational progress for the functional flow equations and the associated scaling functions may lead to a distinction of which type of models for momenta near the Planck scale is viable or not.

到目前为止，我们尚未使用关于波函数重整化或动能项 $Z(\varphi)$ 或 $K(\chi)$ 的定量信息。一旦算出该函数的标度解，就能得到更多重要预言。例如，暴胀期间的慢滚系数 ε 和 η 存在一个用三个标度函数表示的显式公式。由于这些系数直接决定原初宇宙涨落的谱，该预言是可检验的。值得注意的是，宇宙学观测能够证伪带单个标量场的基础标度不变性模型。未来泛函流方程及相关标度函数的计算进展，或许能帮助我们区分哪类普朗克尺度附近动量的模型是自洽的。

For more general renormalizable quantum field theories of gravity, the predictive power is somewhat reduced due to the presence of relevant parameters that can be chosen freely. Still, only a small number of such parameters characterize the possible deviations from the scaling solution. Also for this case we expect substantial additional constraints for cosmology once the scaling form of the kinetial is computed.

对于更一般的可重整化量子引力场论，由于存在可自由选取的相关参数，预言能力会有所降低。但即便如此，偏离标度解的可能情形也仅由少量这类参数刻画。我们预期，即使是这种情况，一旦算出动能项的标度形式，也会为宇宙学带来大量额外约束。

References

参考文献

1. P.A.R. Ade, Z. Ahmed, M. Amiri, D. Barkats, R. Basu Thakur, C.A. Bischoff, D. Beck, J.J. Bock, H. Boenish, E. Bullock et al., Improved constraints on primordial gravitational waves using Planck, WMAP, and BICEP/Keck observations through the 2018 observing season. Phys. Rev. Lett. 127(15), 151301 (2021)

2. Y. Akrami, F. Arroja, M. Ashdown, J. Aumont, C. Baccigalupi, M. Ballardini, A.J. Banday, R.B. Barreiro, N. Bartolo et al., Planck2018 results. Astron. Astrophys. 641, A10 (2020)

3. L. Amendola, Scaling solutions in general nonminimal coupling theories. Phys. Rev. D 60(4), 043501 (1999)

4. L. Amendola, Coupled quintessence. Phys. Rev. D 62(4), 043511 (2000)

5. L. Amendola, M. Baldi, C. Wetterich, Quintessence cosmologies with a growing matter component. Phys. Rev. D 78(2), 023015 (2008)

6. I.G. Avramidy, A.O. Barvinsky, Asymptotic freedom in higher-derivative quantum gravity. Phys. Lett. B 159, 269 (1985)

7. Y. Ayaita, M. Baldi, F. FÜhrer, E. Puchwein, C. Wetterich, Nonlinear growing neutrino cosmology. Phys. Rev. D 93(6), (2016)

8. O. Bertolami, Time dependent cosmological term, Nuovo Cim. B 93, 36-42 (1986)

9. D. Bettoni, J. Rubio, Quintessential inflation: A tale of emergent and broken symmetries. Galaxies 10(1), 22 (2022)

10. D. Blas, M. Shaposhnikov, D. ZenhÅusern, Scale-invariant alternatives to general relativity. Phys. Rev. D 84(4), 044001 (2011)

11. A. Bonanno, A. Eichhorn, H. Gies, J.M. Pawlowski, R. Percacci, M. Reuter, F. Saueressig, G.P. Vacca, Critical reflections on asymptotically safe gravity. Front. Phys. 8, 269 (2020)

12. A. Bonanno, A. Eichhorn, H. Gies, J.M. Pawlowski, R. Percacci, M. Reuter, F. Saueressig, G.P. Vacca, Critical reflections on asymptotically safe gravity (2020)

13. P. Brax, J. Martin, Coupling quintessence to inflation in supergravity. Phys. Rev. D 71(6), 063530 (2005)

14. R.R. Caldwell, R. Dave, P.J. Steinhardt, Cosmological imprint of an energy component with general equation of state. Phys. Rev. Lett. 80(8), 1582-1585 (1998)

15. S. Casas, G.K. Karananas, M. Pauly, J. Rubio, Scale-invariant alternatives to general relativity. III. the inflation-dark energy connection. Phys. Rev. D 99(6), 063512 (2019)

16. S. Casas, M. Pauly, J. Rubio, Higgs-dilaton cosmology: An inflation-dark-energy connection and forecasts for future galaxy surveys. Phys. Rev. D 97(4), 043520 (2018)

17. S. Casas, V. Pettorino, C. Wetterich, Dynamics of neutrino lumps in growing neutrino quintessence. Phys. Rev. D 94(10), 103518 (2016)

18. R. Catena, M. Pietroni, L. Scarabello, Einstein and jordan frames reconciled: a frame-invariant approach to scalar-tensor cosmology. Phys. Rev. D 76(8), 084039 (2007)

19. T. Chiba, Quintessence, the gravitational constant, and gravity. Phys. Rev. D 60(8), 083508 (1999)

20. T. Chiba, M. Yamaguchi, Conformal-frame (in)dependence of cosmological observations in scalar-tensor theory. J. Cosmol. Astropart. Phys. 2013(10), 040-040 (2013)

21. N. Christiansen, D.F. Litim, J.M. Pawlowski, M. Reichert, Asymptotic safety of gravity with matter. Phys. Rev. D 97(10), 106012 (2018)

22. A. Coc, N.J. Nunes, K.A. Olive, J.-P. Uzan, E. Vangioni, Coupled variations of fundamental couplings and primordial nucleosynthesis. Phys. Rev. D 76, 023511 (2007)

23. A. Codello, G. D'Odorico, C. Pagani, R. Percacci, The renormalization group and weyl invariance. Class. Quant. Grav. 30(11), 115015 (2013)

24. E.J. Copeland, A.R. Liddle, D. Wands, Exponential potentials and cosmological scaling solutions. Phys. Rev. D 57, 4686-4690 (1998)

25. T. Damour, G. Esposito-Farese, Tensor-multi-scalar theories of gravitation. Class. Quant. Grav. 9(9), 2093-2176 (1992)

26. T. Damour, F. Piazza, G. Veneziano, Violations of the equivalence principle in a dilaton-runaway scenario. Phys. Rev. D 66(4), (2002)

27. T. Damour, A.M. Polyakov, The string dilation and a least coupling principle. Nucl. Phys. B 423(2-3), 532-558 (1994)

28. J. de Haro, L. Aresté Saló, A review of quintessential inflation (2021)

29. T. Dent, S. Stern, C. Wetterich, Primordial nucleosynthesis as a probe of fundamental physics parameters. Phys. Rev. D 76, 063513 (2007)

30. N. Deruelle, M. Sasaki, Conformal equivalence in classical gravity: the example of "veiled"i general relativity. Cosmol. Quant. Vacuum Zeta Functions 137, 247-260 (2011)

31. R.H. Dicke, Mach's principle and invariance under transformation of units. Phys. Rev. 125, 2163-2167 (1962)

32. K. Dimopoulos, J.W.F. Valle, Modeling quintessential inflation. Astropart. Phys. 18(3), 287- 306 (2002)

33. P. Donà, A. Eichhorn, R. Percacci, Matter matters in asymptotically safe quantum gravity. Phys. Rev. D 89, 084035 (2014)

34. P. Donà, A. Eichhorn, P. Labus, R. Percacci, Asymptotic safety in an interacting system of gravity and scalar matter. Phys. Rev. D 93(4), 044049 (2016)

35. M. Doran, M. Lilley, J. Schwindt, C. Wetterich, Quintessence and the separation of cosmic microwave background peaks. Astrophys. J. 559(2), 501-506 (2001)

36. M. Doran, G. Robbers, Early dark energy cosmologies. J. Cosmol. Astropart. Phys. 2006(06), 026-026 (2006)

37. M. Doran, J.-M. Schwindt, C. Wetterich, Structure formation and the time dependence of quintessence. Phys. Rev. D 64(12), 123520 (2001)

38. D. Dou, R. Percacci, The running gravitational couplings. Class. Quant. Grav. 15, 3449 (1998)

39. G. Dvali, M. Zaldarriaga, Changing α with time: implications for fifth-force-type experiments and quintessence. Phys. Rev. Lett. 88(9), 091303 (2002)

40. A. Eichhorn, Status update: asymptotically safe gravity-matter systems (2022)

41. A. Eichhorn, Y. Hamada, J. Lumma, M. Yamada, Quantum gravity fluctuations flatten the planck-scale higgs potential. Phys. Rev. D 97(8), 086004 (2018)

42. A. Eichhorn, M. Pauly, Constraining power of asymptotic safety for scalar fields. Phys. Rev. D 103(2), 026006 (2021)

43. U. Ellwanger, Flow equations forN point functions and bound states. Z. Phys. C: Particles Fields 62(3), 503-510 (1994)

44. R. Fakir, S. Habib, W. Unruh, Cosmological density perturbations with modified gravity. Astrophys. J. 394, 396 (1992)

45. P.G. Ferreira, C.T. Hill, J. Noller, G.G. Ross, Inflation in a scale-invariant universe. Phys. Rev. D 97(12), 123516 (2018)

46. P.G. Ferreira, C.T. Hill, G.G. Ross, Weyl current, scale-invariant inflation, and planck scale generation. Phys. Rev. D 95(4), 043507 (2017)

47. P.G. Ferreira, M. Joyce, Structure formation with a self-tuning scalar field. Phys. Rev. Lett. 79, 4740-4743 (1997)

48. S. Floerchinger, C. Wetterich, Exact flow equation for composite operators. Phys. Lett. B 680(4), 371-376 (2009)

49. L.H. Ford, Cosmological-constant damping by unstable scalar fields. Phys. Rev. D 35, 2339- 2344 (1987)

50. E.S. Fradkin, A.A. Tseytlin, Renormalizable asymptotically free quantum theory of gravity. Nucl. Phys. B 201, 469 (1982)

51. J.A. Frieman, C.T. Hill, A. Stebbins, I. Waga, Cosmology with ultralight pseudo Nambu-Goldstone bosons. Phys. Rev. Lett. 75(11), 2077-2080 (1995)

52. M. Galante, R. Kallosh, A. Linde, D. Roest, Unity of cosmological inflation attractors. Phys. Rev. Lett. 114(14), 141302 (2015)

53. J. García-Bellido, J. Rubio, M. Shaposhnikov, D. ZenhÅusern, Higgs-dilaton cosmology: from the early to the late universe. Phys. Rev. D 84(12), 123504 (2011)

54. M. Gell-Mann, P. Ramond, R. Slansky, Complex spinors and unified theories. Conf. Proc. C 790927, 315-321 (1979)

55. H. Gies, C. Wetterich, Renormalization flow of bound states. Phys. Rev. D 65(6), 065001 (2002)

56. M. Giovannini, Low-scale quintessential inflation. Phys. Rev. D 67(12), 123512 (2003)

57. D.J. Gross, F. Wilczek, Ultraviolet behavior of non-abelian gauge theories. Phys. Rev. Lett. 30, 1343-1346 (1973)

58. G. Gubitosi, R. Ooijer, C. Ripken, F. Saueressig, Consistent early and late time cosmology from the RG flow of gravity. J. Cosmol. Astropart. Phys. 2018(12), 004-004 (2018)

59. E. Guendelman, R. Herrera, P. Labrana, E. Nissimov, S. Pacheva, Emergent cosmology, inflation and dark energy. Gen. Rel. Grav. 47(2), 10 (2015)

60. A.H. Guth, Inflationary universe: a possible solution to the horizon and flatness problems. Phys. Rev. D 23, 347-356 (1981)

61. A. Gómez-Valent, Z. Zheng, L. Amendola, V. Pettorino, C. Wetterich, Early dark energy in the pre- and postrecombination epochs. Phys. Rev. D 104(8), 083536 (2021)

62. A. Gómez-Valent, Z. Zheng, L. Amendola, C. Wetterich, V. Pettorino, Coupled and uncoupled early dark energy, massive neutrinos and the cosmological tensions (2022)

63. A. Hebecker, C. Wetterich, Natural quintessence? Phys. Lett. B 497(3-4), 281-288 (2001)

64. T. Henz, J.M. Pawlowski, A. Rodigast, C. Wetterich, Dilaton quantum gravity. Phys. Lett. B 727(1-3), 298-302 (2013)

65. T. Henz, J.M. Pawlowski, C. Wetterich, Scaling solutions for dilaton quantum gravity. Phys. Lett. B 769, 105-110 (2017)

66. M. Wali Hossain, R. Myrzakulov, M. Sami, E.N. Saridakis, Class of quintessential inflation models with parameter space consistent with bicep2. Phys. Rev. D 89(12), 123513 (2014)

67. M. Wali Hossain, R. Myrzakulov, M. Sami, E.N. Saridakis, Variable gravity: a suitable framework for quintessential inflation. Phys. Rev. D 90(2), 023512 (2014)

68. M. Wali Hossain, R. Myrzakulov, M. Sami, E.N. Saridakis, Unification of inflation and dark energyáala quintessential inflation. Int. J. Mod. Phys. D 24(05), 1530014 (2015)

69. L. Järv, K. Kannike, L. Marzola, A. Racioppi, M. Raidal, M. Rünkla, M. Saal, H. Veermäe, Frame-independent classification of single-field inflationary models. Phys. Rev. Lett. 118(15), 151302 (2017)

70. L. Järv, P. Kuusk, M. Saal, O. Vilson, Invariant quantities in the scalar-tensor theories of gravitation. Phys. Rev. D 91(2), 024041 (2015)

71. R. Kallosh, A. Linde, D. Roest, Universal attractor for inflation at strong coupling. Phys. Rev. Lett. 112(1), 011303 (2014)

72. S. Karamitsos, A. Pilaftsis, Frame covariant nonminimal multifield inflation. Nucl. Phys. B 927, 219-254 (2018)

73. G.K. Karananas, M. Shaposhnikov, Scale-invariant alternatives to general relativity. II. Dilaton properties. Phys. Rev. D 93(8), (2016)

74. C. Laporte, A.D. Pereira, F. Saueressig, J. Wang, Scalar-tensor theories within asymptotic safety. J. High Energy Phys. 2021(12), 1 (2021)

75. O. Lauscher, M. Reuter, Ultraviolet fixed point and generalized flow equation of quantum gravity. Phys. Rev. D 65, 025013 (2001)

76. G. Lazarides, Q. Shafi, C. Wetterich, Proton lifetime and fermion masses in an so(10) model. Nucl. Phys. B 181(2), 287-300 (1981)

77. A.D. Linde, Chaotic inflation. Phys. Lett. B 129(3), 177-181 (1983)

78. E.V. Linder, The dynamics of quintessence, the quintessence of dynamics. Gen. Relativ. Gravit. 40(2-3), 329-356 (2007)

79. D.F. Litim, Optimized renormalization group flows. Phys. Rev. D 64(10), 105007 (2001)

80. M. Magg, C. Wetterich, Neutrino mass problem and gauge hierarchy. Phys. Lett. B 94(1), 61-64 (1980)

81. P. Minkowski, $\mu \to e\gamma$ at a rate of one out of 109 muon decays? Phys. Lett. B 67(4),421-428 (1977)

82. T.R. Morris, The exact renormalization group and approximate solutions. Int. J. Mod. Phys. A 09(14), 2411-2449 (1994)

83. D.F. Mota, V. Pettorino, G. Robbers, C. Wetterich, Neutrino clustering in growing neutrino quintessence. Phys. Lett. B 663(3), 160-164 (2008)

84. V.F. Mukhanov, G.V. Chibisov, Quantum fluctuations and a nonsingular Universe. JETP Lett. 33, 532-535 (1981)

85. C.M. Müller, G. Schäfer, C. Wetterich, Nucleosynthesis and the variation of fundamental couplings. Phys. Rev. D 70(8), 083504 (2004)

86. G. Narain, R. Percacci, Renormalization group flow in scalar-tensor theories: I. Class. Quant. Grav. 27(7), 075001 (2010)

87. F. Niedermann, M.S. Sloth, New early dark energy. Phys. Rev. D 103(4), L041303 (2021)

88. T. Papenbrock, C. Wetterich, Two-loop results from improved one loop computations. Z. Phys. C Particles Fields 65(3), 519-535 (1995)

89. Á. Pastor-Gutiérrez, J.M. Pawlowski, M. Reichert, The asymptotically safe standard model: from quantum gravity to dynamical chiral symmetry breaking (2022)

90. J.M. Pawlowski, Aspects of the functional renormalisation group. Ann. Phys. 322(12), 2831- 2915 (2007)

91. J.M. Pawlowski, M. Reichert, C. Wetterich, M. Yamada, Higgs scalar potential in asymptotically safe quantum gravity. Phys. Rev. D 99(8), 086010 (2019)

92. P.J.E. Peebles, B. Ratra, Cosmology with a time variable cosmological constant. Astrophys. J. Lett. 325, L17 (1988)

93. P.J.E. Peebles, A. Vilenkin, Quintessential inflation. Phys. Rev. D 59(6), 063505 (1999)

94. M. Peloso, F. Rosati, On the construction of quintessential inflation models. J. High Energy Phys. 1999(12), 026-026 (1999)

95. R. Percacci, Renormalization group flow of weyl invariant dilaton gravity. New J. Phys. 13(12), 125013 (2011)

96. R. Percacci, D. Perini, Constraints on matter from asymptotic safety. Phys. Rev. D 67(8), (2003)

97. R. Percacci, G.P. Vacca, Search of scaling solutions in scalar-tensor gravity. Eur. Phys. J. C 75(5), 188 (2015)

98. A. Platania, From renormalization group flows to cosmology. Front. Phys. 8, 188 (2020)

99. H. David Politzer, Reliable perturbative results for strong interactions? Phys. Rev. Lett. 30, 1346-1349 (1973)

100. M. Postma, M. Volponi, Equivalence of the Einstein and Jordan frames. Phys. Rev. D 90(10), 103516 (2014)

101. V. Poulin, T.L. Smith, T. Karwal, M. Kamionkowski, Early dark energy can resolve the hubble tension. Phys. Rev. Lett. 122(22), 221301 (2019)

102. B. Ratra, P.J.E. Peebles, Cosmological consequences of a rolling homogeneous scalar field. Phys. Rev. D 37, 3406-3427 (1988)

103. M. Reuter, Nonperturbative evolution equation for quantum gravity. Phys. Rev. D 57, 971 (1998)

104. M. Reuter, F. Saueressig, Renormalization group flow of quantum gravity in the Einstein-Hilbert truncation. Phys. Rev. D 65, 065016 (2002)

105. M. Reuter, C. Wetterich, Effective average action for gauge theories and exact evolution equations. Nucl. Phys. B 417, 181-214 (1994)

106. V.A. Rubakov, C. Wetterich, Geodesic (in)completeness in general metric frames (2022)

107. J. Rubio, Higgs inflation. Front. Astron. Space Sci. 5, (2019)

108. J. Rubio, C. Wetterich, Emergent scale symmetry: connecting inflation and dark energy. Phys. Rev. D 96(6), (2017)

109. T.S. Bunch, P.C.W. Davies, Quantum field theory in de sitter space: renormalization by point-splitting. Proc. R. Soc. Lond. 360(A360117-134), 117-134 (1978)

110. S. Sen, C. Wetterich, M. Yamada, Asymptotic freedom and safety in quantum gravity (2021)

111. Q. Shafi, C. Wetterich, Cosmology from higher-dimensional gravity. Phys. Lett. B 129(6), 387-391 (1983)

112. M. Shaposhnikov, I. Tkachev, Quantum scale invariance on the lattice. Phys. Lett. B 675(3- 4), 403-406 (2009)

113. M. Shaposhnikov, D. Zenhäusern, Quantum scale invariance, cosmological constant and hierarchy problem. Phys. Lett. B 671(1), 162-166 (2009)

114. M. Shaposhnikov, D. Zenhäusern, Scale invariance, unimodular gravity and dark energy. Phys. Lett. B 671(1), 187-192 (2009)

115. W. Souma, Non-trivial ultraviolet fixed point in quantum gravity. Prog. Theor. Phys. 102, 181 (1999)

116. B. Spokoiny, Deflationary universe scenario. Phys. Lett. B 315(1-2), 40-45 (1993)

117. A.A. Starobinsky, A new type of isotropic cosmological models without singularity. Phys. Lett. B 91(1), 99-102 (1980)

118. C.F. Steinwachs, A.Y. Kamenshchik, One-loop divergences for gravity nonminimally coupled to a multiplet of scalar fields: calculation in the Jordan frame. I. The main results. Phys. Rev. D 84(2), (2011)

119. K.S. Stelle, Renormalization of higher-derivative quantum gravity. Phys. Rev. D 16, 953 (1977)

120. N. Tetradis, C. Wetterich, Scale dependence of the average potential around the maximum in $\varphi 4$ theories. Nucl. Phys. B 383(1), 197-217 (1992)

121. N. Tetradis, C. Wetterich, Critical exponents from the effective average action. Nucl. Phys. B 422(3), 541-592 (1994)

122. J.-P. Uzan, Cosmological scaling solutions of nonminimally coupled scalar fields. Phys. Rev. D 59(12), 123510 (1999)

123. T.P. Viana, A.R. Liddle, Perturbation evolution in cosmologies with a decaying cosmological constant. Phys. Rev. D 57, 674-684 (1998)

124. S. Weinberg, Ultraviolet Divergences in Quantum Theories of Gravitation, in General Relativity: An Einstein Centenary Survey (Cambridge University Press, Cambridge, United Kingdom and Basel, Switzerland, 1980), p. 790

125. S. Weinberg, The cosmological constant problem. Rev. Mod. Phys. 61, 1-23 (1989)

126. N. Weiss, Possible origins of a small nonzero cosmological constant. Phys. Lett. B 197, 42-44 (1987)

127. C. Wetterich, Cosmologies with variable Newton's "constant". Nucl. Phys. B 302(4), 645- 667 (1988)

128. C. Wetterich, Cosmology and the fate of dilatation symmetry. Nucl. Phys. B 302(4), 668-696 (1988)

129. C. Wetterich, Cosmology with Varying Scales and Couplings, in Strong and Electroweak Matter 2002 (World Scientific, Singapore, 2003)

130. C. Wetterich, Crossover quintessence and cosmological history of fundamental ”constants”. Phys. Lett. B 561(1-2), 10-16 (2003)

131. C. Wetterich, Phenomenological parameterization of quintessence. Phys. Lett. B 594(1-2), 17-22 (2004)

132. C. Wetterich, Growing neutrinos and cosmological selection. Phys. Lett. B 655(5-6), 201- 208 (2007)

133. C. Wetterich, Cosmon inflation. Phys. Lett. B 726(1-3), 15-22 (2013)

134. C. Wetterich, Universe without expansion. Phys. Dark Universe 2(4), 184-187 (2013)

135. C. Wetterich, Eternal Universe. Phys. Rev. D 90(4), 043520 (2014)

136. C. Wetterich, Variable gravity Universe. Phys. Rev. D 89, 024005 (2014)

137. C. Wetterich, Cosmic fluctuations from a quantum effective action. Phys. Rev. D 92(8), 083507 (2015)

138. C. Wetterich, Inflation, quintessence, and the origin of mass. Nucl. Phys. B 897, 111-178 (2015)

139. C. Wetterich, Primordial cosmic fluctuations for variable gravity. J. Cosmol. Astropart. Phys. 2016(05), 041-041 (2016)

140. C. Wetterich, Graviton fluctuations erase the cosmological constant. Phys. Lett. B 773, 6-19 (2017)

141. C. Wetterich, Quantum correlations for the metric. Phys. Rev. D 95, 123525 (2017)

142. C. Wetterich, Gauge invariant flow equation. Nucl. Phys. B 931, 262-282 (2018)

143. C. Wetterich, Quantum scale symmetry (2019)

144. C. Wetterich, Effective scalar potential in asymptotically safe quantum gravity. Universe 7(02), 45 (2020)

145. C. Wetterich, Cosmology from pregeometry. Phys. Rev. D 104(10), 104040 (2021)

146. C. Wetterich, Fundamental scale invariance. Nucl. Phys. B 964, 115326 (2021)

147. C. Wetterich, The great emptiness at the beginning of the Universe. Phys. Lett. B 818, 136355 (2021)

148. C. Wetterich, Primordial flat frame - a new view on inflation. Phys. Rev. D 104(8), 083525 (2021)

149. C. Wetterich, Exact evolution equation for the effective potential. Phys. Lett. B 301(1), 90-94 (1993)

150. C. Wetterich, The Cosmon model for an asymptotically vanishing time dependent cosmological ’constant’. Astron. Astrophys. 301, 321-328 (1995)

151. C. Wetterich, Integrating out gluons in flow equations. Z. Phys. C: Particles Fields 72(1), 139-162 (1996)

152. C. Wetterich, Probing quintessence with time variation of couplings. J. Cosmol. Astropart. Phys. 2003(10), 002-002 (2003)

153. C. Wetterich, Modified Gravity and Coupled Quintessence, in Modifications of Einstein’s Theory of Gravity at Large Distances (Springer International Publishing, Cambridge, United Kingdom and Basel, Switzerland, 2014), pp. 57-95

154. C. Wetterich, M. Yamada, Variable planck mass from the gauge invariant flow equation. Phys. Rev. D 100, 066017 (2019)

155. H. Weyl, Gravitation and electricity. Sitzungsber. Preuss. Akad. Wiss. Berlin (Math. Phys.) 1918, 465 (1918)

156. T. Yanagida, Horizontal gauge symmetry and masses of neutrinos. Conf. Proc. C 7902131, 95-99 (1979)

157. G. Ye, Y.-S. Piao, Is the hubble tension a hint of AdS phase around recombination? Phys. Rev. D 101(8), 083507 (2020)